VOLUME 81 PAPER No. 748

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

JULY, 1955

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

DISCUSSION OF PROCEEDINGS PAPERS

365, 382, 467, 565, 668

HYDRAULICS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

Current discussion of papers sponsored by the Hydraulics Division is presented as follows:

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N.Y.

Discussion of
"EFFECT OF WELL SCREENS ON FLOW INTO WELLS"

by Jack S. Petersen, Carl Rohwer, and M. L. Albertson
(Proc. Paper 365)

WEN-HSIUNG LI,[12] A. M. ASCE.—The authors have conducted a systematic investigation. A simple dimensionless quantity $(C\ L)/D$ was found for determining the hydraulic properties of well screens.

The equation of continuity, Eq. 5a, is incorrect as the discharge Q in the well changes along the direction of flow. The energy equation, Eq. 5b, is also incorrect because, even when frictional loss can be neglected, energy is lost as a result of the impact of the incoming water on the water flowing in the well. That there is a loss of energy caused by impact can be seen from Eq. 12. According to the definition of h_{pz} and Eq. 12,

$$\frac{v^2}{2\,g} + \frac{P}{\gamma} + z = \frac{Q^2}{2\,g\,A^2} + h_{pz} = \tfrac{1}{2}\,(\Delta h_{pz} - \Delta h'_{pz}) + (h_o - \Delta h_{pz}) \quad (18)$$

in which h_o is the piezometric head just outside the screen. The total head varies with Δh_{pz} along the flow in the well. Fortunately, Eqs. 5a and 5b

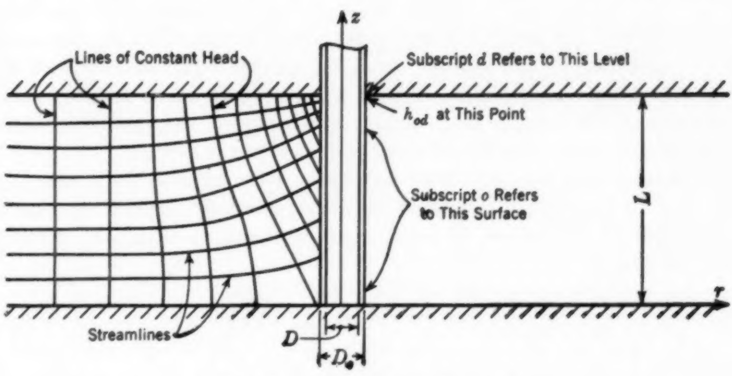

FIG. 19.—SCREEN IN AN AQUIFER

were not used in the investigation. With the assumption that the piezometric head h_o was constant, Eq. 15 was derived; Eq. 15 agrees with observations on screens surrounded by free water (Figs. 2 and 11). The validity of the momentum equation, Eq. 5c, and the orifice equation, Eq. 6, is therefore well demonstrated. However, one might question the validity of the result for screens surrounded by soil in an aquifer. By assuming a constant value for h_o, the authors have obtained a solution which shows that the discharge through the screen is concentrated near the discharge end of the well. If the screen were put in an aquifer, such concentration of flow would necessitate a decrease of piezometric head h_o toward the discharging end, as shown in Fig. 19. The magnitude of the variation of h_o depends on the permeability of the soil among other factors. The higher the permeability of the soil, the less the

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variation of h_o along the screen. In the tests reported by the authors, the free water surrounding the screen is equivalent to a soil with infinite permeability; therefore, h_o is sensibly constant as assumed. When the screen is surrounded by soil of lower permeability, h_o cannot be assumed to be constant.

In the analysis of flow into a well, one must account for the permeability of the soil in the aquifer. If h is the piezometric head at any point in the aquifer, and C_p is the coefficient of permeability of the soil (as defined by the equation for seepage in isotropic, porous materials),

$$q_r = -C_p \frac{\partial h}{\partial r} \qquad (19a)$$

and

$$q_z = -C_p \frac{\partial h}{\partial z} \qquad (19b)$$

in which q_r and q_z are the volumes of flow per unit time per unit cross section of soil in the r-direction and z-direction, respectively (Fig. 19). Both C_p and q have the dimensions of length per unit time. As a result of the continuity of flow in the aquifer,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (20)$$

must be satisfied for axial-symmetric flow.[13] To solve Eq. 20, the conditions at the boundaries of the aquifer must be known. For the case of a screen fully penetrating a confined aquifer (Fig. 19), $\partial h/\partial z$ must be zero at the top and bottom of the aquifer as q_z must vanish there. At great distances from the well, $\partial h/\partial z$ must also be zero as the flow there is radial.

The condition at the remaining boundary (just outside the screen) depends on the flow in the well. If h_d and Q_d are h_{pz} and Q, respectively, at the discharging end of the screen ($z = L$), the momentum equation, according to Eq. 8a, results in

$$h_{pz} = h_d + \frac{Q^2{}_d}{A^2\, g} - \frac{Q^2}{A^2\, g} \qquad (21)$$

in which the subscript d refers to the discharge end of the screen. From a consideration of the continuity of flow across the screen,

$$\frac{dQ}{dz} = \pi\, D_o\, C_p \left(\frac{\partial h}{\partial r}\right)_o \qquad (22)$$

in which D_o is the outside diameter of the screen, $(\partial h/\partial r)_o$ is $\partial h/\partial r$ at $r = D_o/2$ and the subscript o refers to the outside surface of the screen. From a consideration of the continuity of flow in the well, the discharge Q at any point z is

$$Q = Q_d - \int_z^L \frac{dQ}{dz}\, dz = Q_d - \pi\, D_o\, C_p \int_z^L \left(\frac{\partial h}{\partial r}\right)_o dz \qquad (23)$$

[13] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 141.

Substituting the value of Q from Eq. 23 into Eq. 21 results in

$$h_{pz} = h_d + 2\pi \frac{Q_d D_o C_p}{A^2 g} \int_z^L \left(\frac{\partial h}{\partial r}\right)_o dz - \frac{(\pi D_o C_p)^2}{A^2 g}\left[\int_z^L \left(\frac{\partial h}{\partial r}\right)_o dz\right]^2 \quad (24)$$

To relate h_{pz} to the variable h_o along the outside surface of the screen, the use of the orifice equation, Eq. 6, leads to

$$\frac{C_c A_p \pi D}{\sqrt{2 g (h_o - h_{pz})}} = \frac{dQ}{dz} \quad (25)$$

Substituting Eqs. 14d, 22, and 24 into Eq. 25 and simplifying,

$$h_o - h_d - \frac{2\pi Q_d D_o C_p}{A^2 g}\int_z^L \left(\frac{\partial h}{\partial r}\right)_o dz + \frac{(\pi D_o C_p)^2}{A^2 g} \times \left[\int_z^L \left(\frac{\partial h}{\partial r}\right)_o dz\right]^2 - \frac{(8 D_o C_p)^2}{g C^2 D^2}\left(\frac{\partial h}{\partial r}\right)_o^2 = 0 \quad (26)$$

Eq. 26 must be satisfied by the values of h and $\partial h/\partial r$ from Eq. 20 just outside the screen. The solution of Eq. 20 satisfying the boundary condition specified by Eq. 26 can be obtained by relaxation methods.[14] However, some characteristics of the flow can be determined without actual numerical computation.

If h_{od} is the piezometric head in the soil at the discharge end just outside the screen, and $\delta = h_{od} - h_d$ (that is, Δh_{pz} at the discharging end), one must let $\phi = h/\delta$, $R = r/L$, and $Z = z/L$ in order to study the problem in dimensionless form. Eq. 20 can therefore be reduced to

$$\frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R}\frac{\partial \phi}{\partial R} + \frac{\partial^2 \phi}{\partial Z^2} = 0 \quad (27)$$

Eq. 27 must be solved to satisfy the boundary condition at the screen as specified by Eq. 26. Eq. 26 can be rewritten as

$$\phi_o - \phi_{od} + 1 - 2\pi \frac{Q_d D_o C_p}{A^2 g}\int_Z^1 \left(\frac{\partial \phi}{\partial R}\right)_o dZ + \pi^2 \left(\frac{Q_d D_o C_p}{A^2 g}\right)^2 \frac{\delta}{\dfrac{Q^2_d}{A^2 g}} \times \left[\int_Z^1 \left(\frac{\partial \phi}{\partial R}\right)_o dZ\right]^2 - 4\pi^2\left(\frac{D}{C L}\right)^2 \left(\frac{Q_d D_o C_p}{A^2 g}\right)^2 \frac{\delta}{\dfrac{Q^2_d}{A^2 g}}\left(\frac{\partial \phi}{\partial R}\right)_o^2 = 0 \quad (28)$$

in which the subscript *od* refers to the discharge end just outside the screen. It can be seen that, given the geometrical shape of the aquifer (that is, D_o/L) and given the coefficients in Eq. 28, a unique solution of Eq. 28 can be obtained. That is, the loss coefficient equals

$$\frac{\delta}{\dfrac{Q^2_d}{A^2 g}} = f\left(\frac{C L}{D}, \frac{Q_d D_o C_p}{A^2 g}, \frac{L}{D_o}\right) \quad (29)$$

[14] "Relaxation Methods in Theoretical Physics," by R. V. Southwell, Oxford University Press, Oxford England, 1946.

The actual value of ϕ_{od} is immaterial as it represents a constant head for the entire aquifer. In order to have some insight into the effect of C_p on the loss coefficient in Eq. 29, let

$$m \frac{Q_d}{L} = \pi D_o C_p \left(\frac{\partial h}{\partial r}\right)_o \quad \text{(30)}$$

in which m is the ratio of the rate of flow through the screen at any point to the mean rate of flow through the screen. With this coefficient m, Eq. 26 can be reduced to

$$\frac{h_o - h_{od}}{\frac{Q^2_d}{A^2 g}} + \frac{\delta}{\frac{Q^2_d}{A^2 g}} - 2 \int_Z^1 m\, dZ + \left(\int_Z^1 m\, dZ\right)^2 - 4 \left(\frac{D}{C L}\right)^2 m^2 = 0 \quad \text{(31)}$$

At the discharging end of the screen, $Z = 1$, $h_o = h_{od}$, and $m = m_o$. Thus,

$$\frac{\delta}{\frac{Q^2_d}{A^2 g}} = 4 \left(\frac{D}{C L}\right)^2 m^2_o \quad \text{(32)}$$

Thus, the loss coefficient is dependent on $\frac{D\, m_o}{C L}$, in which m_o is the value of m at the discharging end of the screen. The value of m_o depends, of course, on the independent variables in Eq. 29. In the case of a fully penetrating well, the two extreme values of m_o are obtained from the case of constant h_o and the case of uniform discharge through the screen. Both of these cases are never realized in the field, but they serve as the limits to the actual cases. A sensibly constant h_o is obtained with very permeable soils which offer almost no resistance to the concentration of the discharge at the discharge end of the screen; the authors' result is an approximate solution for such cases. On the other hand, an approximately uniform distribution of discharge through the screen is obtained with soils of low permeability which offer high resistance to the concentration of the discharge. An approximate solution for such cases can be obtained by setting m_o equal to unity. Thus, for small values of $\frac{Q_d D_o C_p}{A^2 g}$, the loss coefficient can be approximated as

$$\frac{\delta}{\frac{Q^2_d}{A^2 g}} = 4 \left(\frac{D}{C L}\right)^2 \quad \text{(33)}$$

When Eqs. 15 and 33 are plotted in Fig. 20, it can be seen that the two curves coincide for $(C L)/D$ less than unity. This means that, for all cases in which $(C L)/D$ is less than unity, both h_o and the discharge through the screen are approximately constant. For $(C L)/D$ larger than unity, the curve for a given screen may lie anywhere between these two curves. The value of $(C L)/D$ for a minimum value of the loss coefficient must be larger than 6, and the minimum loss coefficient must be less than unity.

Additional factors may also influence the value of m_o, and therefore the magnitude of the loss coefficient. For partly penetrating wells, there is a tendency for the discharge to concentrate near the bottom of the screen,[15] thus reducing the value of m_o. When the screen is surrounded by gravel, the analysis previously presented must be modified as Eq. 22 is not true when there is a redistribution of the discharge through the gravel layer. The actual problem is complicated. The authors' result gives a loss through the screen

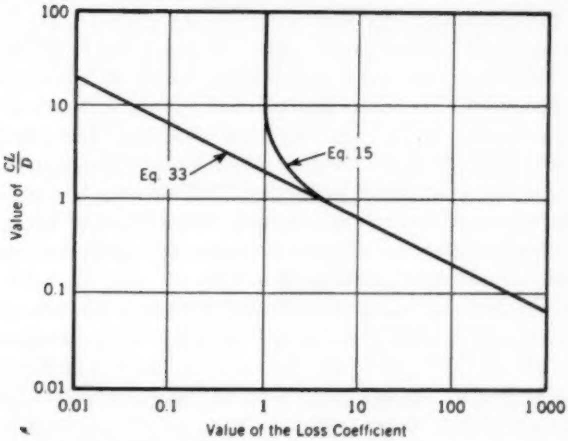

FIG. 20.—THE RELATIONSHIP BETWEEN $(CL)/D$ AND THE LOSS COEFFICIENT

which is the greatest loss that can possibly occur and is, therefore, of great practical value.

Another point of interest in this subject should also be mentioned. The condition of constant h_o or uniform distribution of discharge through the screen is never realized in nature. Thus, even for fully penetrating wells, the flow in the aquifer can never be truly radial. This fact is not generally recognized in the study of flow into wells.

ARTHUR L. COLLINS,[16] A. M. ASCE.—In the "Synopsis" it is stated:

> "The objective of the investigation was to establish criteria which could be used to aid in the selection of well screens to meet the varied conditions found throughout the United States."

Another interesting statement is contained in the "Conclusions": "For the criteria to be of greatest practical value to the well driller, a large extension of the available coefficients is necessary." It would be of interest to know to what extent the laboratory tests should be extended. Also, what value should the well driller place on the test information?

In their investigation the authors have anticipated the construction of the gravel-packed well. This type of construction necessitates the boring of a

[15] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 269.

[16] Cons. Engr., Berkeley, Calif.

large-diameter well which is protected temporarily by a large casing and then by the installation of a smaller casing with perforations. This installation is followed by the placing of selected gravel between the casings and finally by the removal of the outside casing.

The reasoning behind the gravel-packed well is correct, but in practice this type of construction is unreasonably expensive. In contrast, the well driller who is familiar with the substrata in which he is drilling has probably learned to construct a well which will satisfy the requirements of the user at a fraction of the cost of the made-to-order well with packed gravel. For example, the many wells constructed for irrigation purposes are principally located in the alluvial fills of valleys and are not founded in extensive rock formation. There must also be present a clay or hardpan stratum on top of the sand and gravel through which the water must pass. It is believed that the average well and the large-quantity well have clear-water pockets which exist at the top of the sand stratum and are bridged by a clay roof. The well, if normal, will have part of the perforations entirely unobstructed. As a result of this unobstructed condition, there is insufficient outside pressure to cause the water to flow through the partly obstructed perforations.

The well driller uses the factory-perforated casing, or he makes the perforations at the proper depth after the casing is in place with a perforating instrument. By the use of these methods he is able to make a well which is equal to, or perhaps superior to, the gravel-packed well at a fraction of the cost.

In order to explain the water-pocket theory, a description of a well system without any perforations will be presented. The well with large perforations made after the casing is in place and opposite the assumed pocket is virtually a screenless well.

There are many localities where wells can be constructed without the use of screens. Such a location would be one where there are thick clay deposits separated by sand and gravel strata of a few feet in thickness. The clay cap must be compact enough to bridge the sand if a part of the sand is removed. Also, the strata must be below the water level in the well at all times. A well that will pump sand to any extent can be assumed to contain a pool in which the clear water collects.

The subject of well development first interested the writer in 1910 when he learned that in California ample irrigating water could be obtained from bored wells or augered wells. It was also at this time that the deep-well turbine became available. This turbine provided the solution to the problem of drawing the water from a deep well without providing deep pits for the centrifugal pump or for other cumbersome equipment.

The substrata of the valley floor from Bakersfield to Corning in the San Joaquin and Sacramento valleys of California (a distance of approximately 400 miles) follows this general pattern (clay deposits separated by sand and gravel strata). In this area there has been little change in the type of well and casing that has been used from 1910 to 1954. Both the slotted section made in the shop and the slots made in the field after the well is constructed are used. From 1910 until welding equipment became available in the field the drillers favored 4-ft-long, double-casing sections. The sections, having

slightly different diameters, were telescoped with the joints made at the mid-section. The pipe presently used (1954) is the 4-ft section of heavy-gage steel with welded collars.

The strainerless well was patented by the writer on April 28, 1914. The idea originated from a clue discovered in a locality where an unusually large supply of water could be obtained from a few feet of sand and gravel. In these wells the contractor ended the casing some distance above the gravel stratum and used no strainer. Elaborating on this idea and applying it to other areas where the sands were finer, it was observed that, when the well was pumped heavily, sand would be emitted and afterward the water would become clear. This action results because the water itself flushes out a percolating area until there is a balance between the water velocity and the sand. It was also assumed that the percolating space extended radially along the roof of the sand stratum from the well to form a sheet of thin depth.

Therefore, if the casing is extended to the floor of the sand stratum, the sand will be moved downward by gravity, and the void area of the well will be increased. This increase, however, results from the removal of too much sand. If the strainer is opposite the sand, the well is improved by heavily perforating the casing near the top of the sand stratum. The strainer adds little to the well efficiency.

After reaching these conclusions concerning the open-bottom well, the writer conceived the idea of constructing a well with a 12-in. casing or a 14-in. casing which was terminated at the first sand stratum. Then a second string of 10-in. casings was extended into the second stratum. The 10-in. casing was then cut off near the end of the first stratum—thus leaving an annular space through which the water from the first stratum was taken into the casing to join the water from the lower stratum. In this way it was possible to extend the open-bottom-well principle to several strata in the well.

It is of interest to note that the writer was a supervising engineer and not an experienced well driller when the theory of the open-bottom well was deduced. However, not long after the patent was granted, a professional well driller sought a license to use the patent. Prior to that date the well driller had constructed fifty wells according to this principle, and only when he applied for a patent did he learn that one had been previously obtained.

As a check on the work of this well driller, an agent for a manufacturer who furnished most of the well casing for several drillers operating in the area observed the wells made by this particular driller. The agent concluded that the driller's wells produced 50% more water (with no sand pumping) than did other wells.

Because of the sight-unseen conditions in a well and the methods of construction which differ among well drillers, it will always be difficult to obtain data—such as the size of the casing, the area of the perforations, and the size of the sand grains—which will enable the engineer to predetermine the performance of a well.

Mohammad Nazir[17] and Nazir Ahmad.[18]—A solution to the problem of excessive head loss through screens (which are in general use (1954) in the Punjab Province of Pakistan) has been presented by the authors. Since the inception of the use of ground water, engineers in the Punjab Province have used a screen made of ⅛-in.-thick brass sheets in which horizontal slits, 1 in. long and of varying widths (from 0.010 in. to 0.014 in.) are cut. These slits are sharp on the outside and expand toward the inside at an angle of approximately 30°. The distance between adjacent slits is ⅛ in. In order to maintain sufficient strength in the strainer, a ¾-in. space is left between two sets of 1-in. slits. Thus, the open area of these screens is from 6% to 10% of the total surface area.

The prevalent aquifer formation in a part of the Punjab Province is composed of medium sand; therefore, the coarsest formation has a mean size of between 0.30 mm and 0.45 mm. The screens are usually placed in medium sand which has a mean diameter of between 0.25 mm and 0.45 mm and an effective size of between 0.15 mm and 0.25 mm. The sand has a high coefficient of uniformity—between 1.6 to 2.5—and the usual diameter of the brass screen

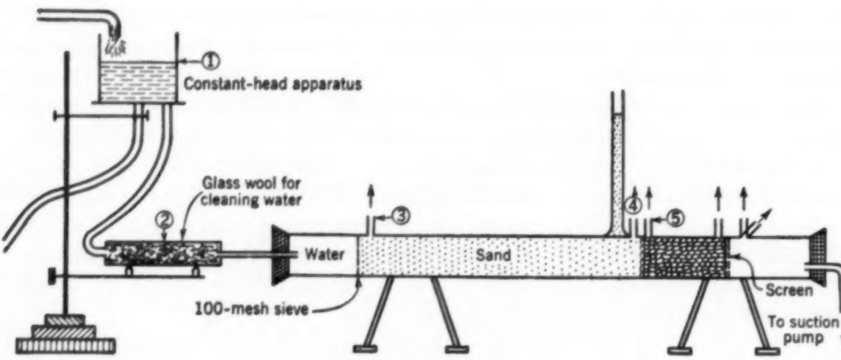

Fig. 21.—Test Setup

is between 7½ in. and 9 in. A length of approximately 150 ft is considered sufficient to pump 2 cu ft per sec. In order to exclude the low-yield region and any saline water, a blind pipe with a length of from 40 ft to 100 ft is normally installed below the natural surface.

Several pumping tests have been performed in regions where the water table lies from 3 ft to 10 ft below the natural surface. A study has also been made of the accompanying lowering of the water table. It has been noted that, even at a distance of 2 ft from a well screen, the lowering of the water table has seldom exceeded 20% of the pumping level in the well. The lowering observed at a distance of 1,000 ft from the well has measured from ½ ft to 1 ft. This small lowering has been attributed to: (1) The formation of free surface; (2) the head loss from the formation into the shrouding which envelopes the screen; and (3) the head loss between the gravel shrouding and the screen.

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[18] Head, Physics Dept., Irrig. Research Inst., Punjab, Lahore, Pakistan.

If this lowering is attributed to the concept of a free surface, the existence of a long blind pipe and the absence of a discharge face are arguments against this concept. It can be argued that the small lowering is caused by the greater yield of the formation. However, the pumping level in the well contradicts this argument.

Some laboratory experiments have been performed to study the head loss between the formation and the gravel and that between the gravel and the screen. Assuming the flow in the screen region to be horizontal, the experimental arrangement shown in Fig. 21 was devised. A supply of air-free and filtered water was made available on the inflow side, and on the exit side suction up to 20 ft was applied. Small outlets were provided at points 1, 2, 3, 4, and 5 to record pressure by mercury manometers. The screen studied had four different slit sizes—0.014 in., 0.020 in., 0.030 in., and 0.040 in.—with open areas equal to 7.65%, 11.0%, 13.4%, and 19.3% of the total surface area, respectively. The head losses between the gravel and the screen and between the sand and the gravel and the observed discharge are shown in Table 3.

TABLE 3.—THE EFFECTS OF INCREASE OF SLIT SIZE ON DISCHARGE AND HEAD LOSS

Suction head, in feet	Discharge, in Cubic Centimeters per Minute				Head Loss, in Feet							
	Slit Size, in Inches											
	0.014	0.020	0.030	0.040	0.014		0.020		0.030		0.040	
	Open area, in percentage of total area				Between sand and gravel	Caused by screen	Between sand and gravel	Caused by screen	Between sand and gravel	Caused by screen	Between sand and gravel	Caused by screen
	7.65	11.01	13.4	19.3								
5	224	238	260	266	0.62	0.17	0.58	0.17	0.46	0.09	0.37	0.08
7	302	318	346	354	1.22	0.20	0.94	0.20	0.76	0.12	0.64	0.10
10	421	436	476	485	1.82	0.23	1.60	0.23	1.40	0.16	1.26	0.15
12	504	519	560	568	2.18	0.26	2.00	0.25	1.88	0.20	1.70	0.18
15	636	640	692	710	2.65	0.28	2.45	0.28	2.50	0.15	2.32	0.30
17	720	722	784	808	2.90	0.30	2.70	0.30	2.81	0.31	2.74	0.30

A study of Table 3 reveals that the head loss between the screen and the gravel is nearly constant, the variation being from 0.1 ft to 0.3 ft with suction head varying from 5 ft to 17 ft. The head loss between the formation and the gravel is, however, significant, as it is approximately ten times as great as that between the gravel and the screen. Similarly, the discharge with a given suction head was the highest for the screen with the widest slits. Experiments with wider slits and open area greater than 20% are in progress (1954), but the preliminary observations have shown that screens with slits wider than 0.040 in. may not increase the discharge under a given suction head.

Although there is no question about the mathematical derivation of Eq. 15, it does seem that in the authors' range of application, laminar flow should exist

and there should be no movement of particles. Therefore, the effect of viscosity cannot be eliminated. Similarly, the statement that the factor $(C\,L)/D$ should always be greater than 6 to reduce the screen loss to a minimum seems difficult to comprehend. From the writers' experience in Pakistan, D does not exceed 9.0 in., L can be as great as 150 ft., and the value of C (from Eq. 14d) can be much greater than 30. Therefore, the factor $(C\,L)/D$ will always be greater than 6, and the loss will be minimum regardless of the water pumped out. However, a consideration of this point does not explain the high pumping level, high yield, and attendant slight lowering of the water table in Pakistan.

MATTHEW I. RORABAUGH,[19] A. M. ASCE.—The experimental data presented by Messrs. Petersen, Rohwer, and Albertson are a valuable contribution to the hydraulics of wells. Much speculation and many claims have been made concerning the magnitude of head losses through well screens. The data presented should do much toward clarifying some of the problems involved in screen design and in the selection of well screens.

The authors are to be complimented on their work considering the difficulties inherent in such a study. The economical construction of a suitable model which will be hydraulically equivalent to the prototype is particularly difficult. Small-scale models of screens are costly items. Other items also add to the cost of research and to the sensitivity of the necessary physical observations.

One of the basic assumptions in the theoretical development is that the piezometric head on the outside of the screen is constant over the entire outside surface of the screen. In the case of a gravel envelope, this condition was not met, as is implied in Fig. 8. The authors considered this variation to be relatively small and concluded that the status of the investigation did not warrant the use of a correction for the variation. The effects shown in Fig. 8 result, in part, from the fact that the screen (2 ft long) does not fully penetrate the saturated thickness (5 ft). Flow enters the gravel pack over this 5-ft length and travels through the gravel to discharge through the 2-ft screen. Flow lines converge radially in the horizontal plane and also converge in the vertical plane. The head losses in Fig. 8 are taken horizontally through the gravel pack. The shape of the curves in Fig. 8 is partly controlled by the position of the screen in relation to the boundaries—the bottom of the tank and the water surface in the tank.

The effects of the boundary conditions on flow toward the screen in the laboratory structure are related to the data in Figs. 16, 17, and 18. From reference to the test apparatus shown in Fig. 6 and to the tests on screens without a surrounding grave pack, it should be evident that, as A_p approaches 100%, the horizontal component of velocity at the screen varies from approximately zero at the water surface to a maximum near the base of the screen. This generalization can be visualized easily by considering the flow net that would be developed between the overflow rim and the outflow pipe shown in Fig. 6. That is, for the condition of no screen or screens having a low friction loss, head-distribution curves similar in shape to those in Figs. 16, 17, and 18 would result. These curves would be based on the flow pattern in the tank

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and would be controlled by the tank boundaries and the position of the inflow and outflow pipes. Item 4 in the "Conclusions," apparently based on those data in Fig. 17, applies to the test apparatus used. However, considering the effects of boundary conditions and of the partial penetration introduced by the addition of a gravel pack 9 in. thick (as demonstrated in Fig. 8), it is doubtful whether this conclusion can be applied to prototype structures.

The investigation deals with head losses in a very small zone in and near the screen. This is only a small part of the problem of total drawdown in a well. Before the screen data are used to form conclusions for general well design, it would be desirable to consider comprehensively the head distribution in and about the pumping well.

If an artesian gravel bed, 4 ft thick, having a permeability of 2,500 gal per day per sq ft and a storage coefficient of 0.003, is fully penetrated by a 4-ft-long, 3-in.-diameter, A-d-type, well screen, and if pumping is conducted at rate of 0.125 cu ft per sec, the conditions approximate those shown in Fig. 17. For a pumping time of 1 hr the drawdown in the well, computed by the nonequilibrium formula developed[20] by C. V. Theis, is approximately 10 ft, if turbulent losses in the aquifer are not considered.[21] According to this conclusion (item 4) the most efficient well screen would have to penetrate only 1 ft of the 4-ft aquifer (Fig. 17). Using the Kozeny[22] equation for partial penetration, the drawdown at the well is approximately 15 ft rather than 10 ft. A plot of head distribution about a partly penetrating well is given by M. Muskat.[23] Thus, the selection of an efficient well screen for this example (by use of item 4 in the "Conclusions") results in a crowding of the flow lines outside the well and inefficient use of the aquifer. Although the head loss for the 1-ft screen and the 4-ft screen will be almost the same when tested in a tank, the drawdown will differ by approximately 5 ft when installed in an aquifer of the type used in the example.

The permeability of the aquifer is an important item in examining the various factors influencing well design. In an aquifer having a very high permeability, the aquifer losses will be small, and attention to a modified form of item 4 in the "Conclusions" could be justified by a consideration of the head distribution in the aquifer. However, in most of the unconsolidated aquifers tested in the United States, the usual operating head loss is measured in feet and in tens of feet. For these wells, a screen loss of a few tenths of a foot is small in proportion to the total loss, and item 4 in the "Conclusions" would not form an important part of the economies in the entire flow system.

It is noted that the rates of flow used in the studies were, in general, higher than those in most producing wells. Rates in the tests, for example, were 0.125 cu ft per sec through 1 ft of a 3-in. screen and 1 cu ft per sec through 2 ft of a 12-in. screen. With these rates, only a few tenths of a foot of head loss

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[20] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by C. V. Theis, *Transactions*, Am. Geophysical Union, Vol. 16, 1935, p. 520.

[21] "Graphical and Theoretical Analysis of Step-Drawdown Test of Artesian Well," by M. I. Rorabaugh, *Proceedings-Separate No. 362*, ASCE, December, 1953.

[22] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, J. W. Edwards, Inc., Ann Arbor, Mich., 1946, p. 274.

[23] *Ibid.*, p. 270.

was measured. In a field installation, using 20 ft of a 12-in. screen and producing 500 gal per min, losses through the screen become small when compared to other losses.

This investigation should do much to clear up misunderstandings between owners, drilling contractors, and screen manufacturers as to the extent of the head losses of screens. The data demonstrate that losses in most commercial screens are small, and that the screen manufacturers have done well in designing efficient screens.

DEAN F. PETERSON, JR.,[24] M. ASCE.—A most interesting experimental verification of a theoretical hypothesis in fluid mechanics has been presented; experimental results are seldom as satisfactory as in this case. The authors are to be complimented on their application of the basic laws of hydrodynamics to the development of this hypothesis and the manner in which they organized their experimental program to check it.

In the tests that omitted a gravel pack, the outside piezometric head was essentially constant. This constancy could not be true of a well in a sand formation or in a gravel formation. The screen phenomenon would undoubtedly cause the streamlines approaching the well to converge near the discharge end of the screen and would thus cause pressures outside the casing to be lowest near the discharge end. This would tend to overcome, to some extent, the effects of the screen which were reported by the authors. The foregoing effect would certainly be secondary, and it is expected that the authors' analysis will still give useful quantitative results.

Most welcome would be a solution to the problem of the hydraulics of a well in an aquifer which would take into account the screen effects suggested by the authors. All solutions of the problem of well hydraulics, such as that offered by Jules Dupuit,[25] are based on the assumption of equal piezometric head throughout the length of the well.

Another interesting problem is the effect of a pump intake located within the length of the screen. The velocities inside the screen would converge from both directions toward the pump intake. One would thus infer that, by proper placement of the pump intake, a "double effective" length of screen might be developed—one length above the intake and one below. For the most effective operation, it would appear that the pump intake would need to be set above the bottom of the screen a distance such that $(C\,L)/D$ is greater than 6 in order that the screen might be unrestricted by pump bowls and columns throughout the effective length. In addition, there would be an effective distance above the pump intake of $(C\,L)/D$ equal to 6. For this latter case, D would need to be corrected to allow for the area occupied by the pipe columns and bowls. Experiments on an effective screen length using actual pump installations would be enlightening; perhaps this might be done by the use of a small-scale pump and screen.

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[25] "Études théoretiques et pratiques sur le mouvement des eaux," by Jules Dupuit, Paris, France, 2d Ed., 1863.

G. COHEN DE LARA.[26]—The development of ground-water resources in Algeria and Tunisia necessitated a general study of friction losses in well screens and in the surrounding aquifer. This study, begun in 1949, resembles in part that performed by Messrs. Petersen, Rohwer, and Albertson. On the whole, these results agree with those of the authors, but some of their conclusions appear to be incomplete.

The problem was approached by determining the theoretical laws of continuous flow through screens, with and without porous material surrounding them. In order to support the theory, air-flow tests (without surrounding material) were performed according to the Reynolds similarity. The tests

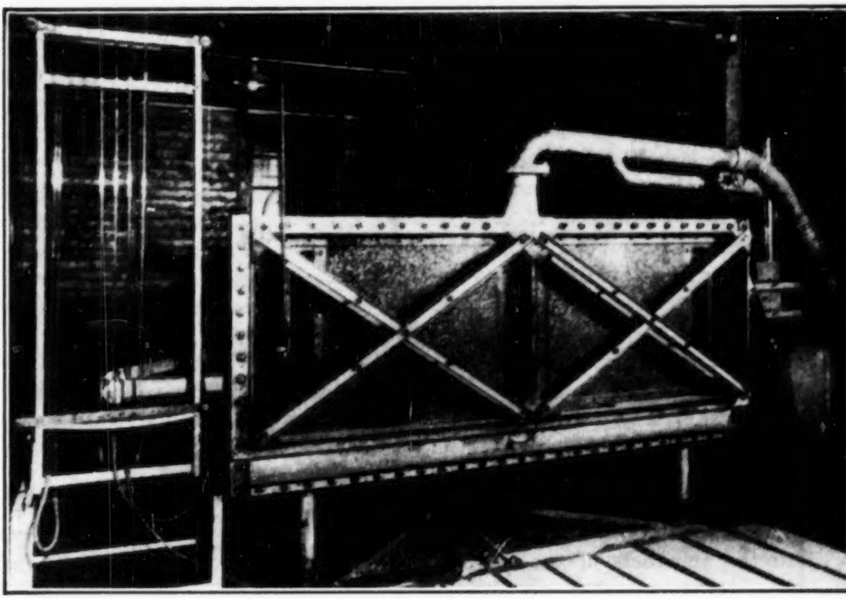

FIG. 22.—SEMICIRCULAR MODEL

with surrounding material were conducted with water on the semicircular model shown in Fig. 22. The results of these latter tests have not been published (as of 1954). The purpose of these tests is similar to the purpose of those performed by the authors, although different methods were used.

Theoretical Computation and Experimental Research for a Screen Surrounded Only by a Liquid.—The theoretical computation is similar to that offered[27] by P. Arnaud, and the experimental measurements accurately confirm the basic assumptions. It is agreed that the experimental determination of a coefficient C_s (which varies according to the type of commercial screen used) allows one to apply the theoretical computation to commercial screens, to which the basic assumptions used in the computation do not always apply.

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[26] Engr., Laboratoire Dauphinois d'Hydraulique, Grenoble, France.

[27] "Pertes de Charge dans les Crépines de Puits de Pompage," by P. Arnaud, *Troisième Journées de l'Hydraulique*, Paris, France, 1954.

Well Screen Surrounded by Gravel.—During the tests on a circular model, it was found (as also by the authors) that the pressure on the exterior wall of the screen was not constant. For a louvre-type screen (200 mm in diameter and 70 cm in length) with high discharges, there resulted curves similar to those shown in Fig. 8. It is not known whether the trend of these curves is maintained for small discharges as the resulting pressure differences were too small to be measured.

In the theoretical computation, it is supposed that the pressure on the exterior wall is constant. The authors have tried to avoid the difficulty by determining a coefficient, C_s, similar to the one used for commercial screens surrounded only by liquid. The head loss, Δh, entering in the dimensionless number, $\Delta h \Big/ \frac{V^2}{2g}$, was taken as the difference in pressure between the static pressure intake in the interior of the screen and the static pressure intake half way along the screen, between the gravel and the screen—that is, where there is a minimum static pressure for the given discharge. Based on this definition, the authors determined several values for C_s, which appears as a function of the opening dimensions and size of gravel used for a given type of screen. It is implied by the authors that C_s is applicable regardless of the voids ratio of the surrounding soil and of the thickness of the gravel. This result seems incomplete as a change in the surrounding soil thickness or of its voids ratio will entail a variation of Δh—therefore of C_s—even for the same discharge using the same apparatus.

Furthermore, a second theoretical computation has been established[27] by Mr. Arnaud for a screen surrounded by gravel in which head losses are assumed to be laminar. This computation has been verified by tests on the semicircular model. The results of this theoretical and experimental work show that the minimum length, L, of the screen—beyond which $\Delta H \Big/ \frac{V^2}{2g}$ remains constant regardless of further increase of the screen length—depends on the permeability, K, and the thickness of the surrounding material. The resulting formula is

$$\frac{\Delta H}{\frac{V^2}{2g}} = f\left(\frac{L\,K}{\log\frac{t}{r}}\,\frac{\sqrt{h}}{C_{qs}}\right) \qquad (34)$$

in which ΔH is the drop in head across the screen; K denotes the permeability of the soil; t is the thickness of the surrounding soil; r represents the screen radius; h is the total head loss through the screen, gravel, and aquifer; and C_{qs} denotes the discharge coefficient for the screen slots. The surrounding material also changes the contraction coefficient for the openings and the percentage of open area of the screen. These two changes depend on the relative dimensions of screen openings and gravel. The over-all effect of the surrounding gravel on the screen, characterized by the discharge coefficient, C_{qs} (Eq. 34), was determined by a comparison—at constant discharge—of head losses in a screen and filter of given thickness obtained in the two following cases: (1) A well screen surrounded by fine wire gauze, with a percentage of open area

higher than that of the surrounding material, and (2) an ordinary well screen, without wire gauze. The discharge coefficient, C_{qs}, could thus be experimentally determined as a function of the mean diameter of the surrounding grains.

The values of C_s determined by Messrs. Petersen, Rohwer, and Albertson therefore apply only to the conditions on their model—that is, for the filter thickness and voids ratio used by them.

TABLE 4.—RELATIONSHIP AMONG SCREEN COEFFICIENT, GRAVEL SIZE, AND SLOT WIDTH [a]

Screen coefficient (1)	Mean diameter of gravel, in inches (2)	Slot width, in inches (3)	Col. 2 ÷ Col. 3 (4)	Mean value of Col. 4 (5)
0.3	0.075	1/16	1.2	1.23
	0.16	1/8	1.28	
	0.23	3/16	1.23	
	0.3	1/4	1.2	
0.4	0.12	1/16	1.92	1.9
	0.24	1/8	1.92	
	0.36	3/16	1.92	
	0.44	1/4	1.75	
0.5	0.17	1/16	2.71	2.75
	0.34	1/8	2.71	
	0.55	3/16	2.94	
	0.68	1/4	2.71	
0.6	0.24	1/16	3.84	4.7
	0.55	1/8	4.9	
	0.95	3/16	4.9	

[a] Values obtained from Fig. 13.

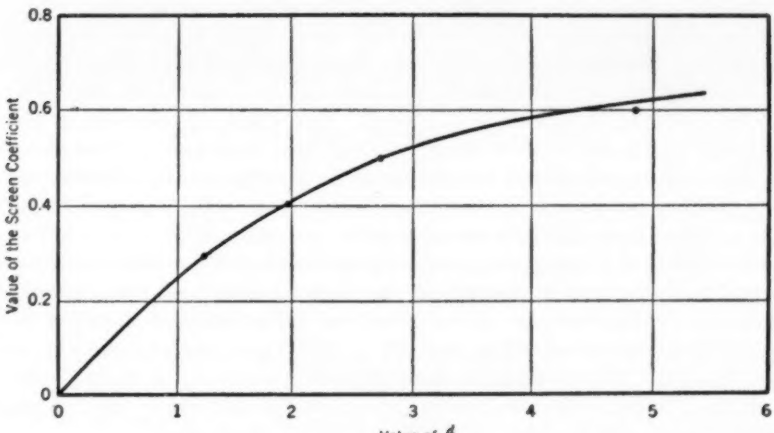

FIG. 23.—VARIATION OF SCREEN COEFFICIENT WITH GRAVEL SIZE AND SLOT WIDTH

The Variation of the Value of the Screen Coefficient.—Fig. 13 represents the variation of C_s as a function of the size of surrounding gravel for different widths of type B screen openings. It appears from Fig. 13 that a geometric similitude does not exist and that different values of C_s can be found for a given ratio of d (diameter of gravel) to s (slot size). Using the authors' results, it was possible to see that all the curves of Fig. 13 can be reduced to a single curve,

with the variation of C_s considered to be a function of d/s (Table 4 and Fig. 23). Geometric similitude is thus obtained for type B screens. It would be interesting to make the same observations for screens of other types.

Practical Application.—The practical application of the results obtained in this study requires a knowledge of the hydro-geologic variables entering into Eq. 34 and of the permeability of the surrounding aquifer. The composition of a natural or artificial filter, intended to prevent sand particles from entering the well and to diminish the loss of head in the immediate vicinity of the well, complicates the problem. The minimum length that can be given to a screen will depend on the thickness of the natural or artificial filter, the hydro-geologic conditions, and the permeability of the filter and aquifer. If it were possible to determine C_s as a function of these variables and to determine the effect of C_s on C, it would be possible to apply Conclusion 1 of the paper.

In view of the difficulties that must be overcome, it can be seen how much knowledge and intuition are needed to determine accurately the minimum length of a well screen. Nevertheless, an examination of the combination of screen and soil shows that a great part of the head loss occurs in the immediate vicinity of the screen because of the convergence of flow. It is then possible to determine an "effective radius of influence," which can be defined as the radius of the zone within which 95% of the total head loss in the combination screen-aquifer occurs. Knowing the soil composition and assuming maximum consolidation, it is possible to determine the minimum economical length of the screen to be installed in a well.

Gérard Tison, Jr.[28].—At the hydraulic laboratory of the University of Ghent in Belgium, some investigations have been made concerning the resistance to a flow of water through a special type of well screen. The screen was made of steel with large elongated holes and was surrounded by an envelope (of gravel or sand) 14 mm (0.55 in.) thick. The grains of this sand or gravel were bound by plastic matter which was kept to a minimum so that the permeability of the envelope was held as nearly as possible to the natural permeability. The grain size of the sand used in the test sample ranged from 0.7 mm to 1.2 mm (from 0.028 in. to 0.047 in.).

The methods of investigation were slightly different from those used by the authors. A well-screen element 0.86 m (2.62 ft) long and with a 0.076-m (1.93 in.) inside diameter was placed along the horizontal axis of a steel cylindrical tank the diameter of which was 0.90 m (2.74 ft) and the length of which was 1.25 m (3.81 ft). This cylindrical tank (Fig. 24) was closed at its two ends by steel sheets. A 2-in.-diameter pipe which was fitted into one of the end plates of the cylinder was the outlet for the flow through the screen.

The cylindrical tank was connected with a water supply. The pressure in the tank was maintained at a constant magnitude by a valve on the supply pipe. Under the effect of that pressure the water flowed through the well screen. Outside the well screen there was hydrostatic pressure. Inside the well screen the value of the pressure was a maximum at the closed end and a minimum at the outlet pipe.

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Investigations concerning filtration have shown that movement is laminar so long as

$$\mathbf{R} = \frac{V_f\, d}{\nu} < 5 \quad (35)$$

in which V_f, the filtration velocity, is

$$V_f = \frac{Q}{A_f} \quad (36)$$

R represents the Reynolds number, d is the effective grain size, ν denotes the kinematic viscosity, and A_f is the area of the filtration surface.

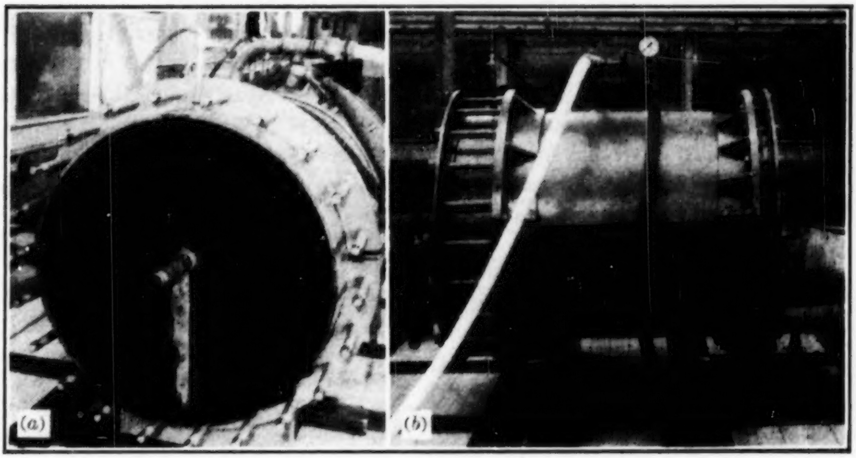

FIG. 24.—HORIZONTAL EXPERIMENTAL WELL SCREEN

Several of the investigations at the University of Ghent satisfied the condition represented by Eq. 35. It is known that in this case the discharge is proportional to the difference of piezometric head between the inside and the outside of the screen. Applying the momentum equation to the flow within the pipe between the closed section (0) through which no flow passes and any section (1) of that pipe,

$$A\gamma\,(h_{pz,0} - h_{pz,1}) = \frac{\rho\, Q^2}{A} \quad (37)$$

As the piezometric head is constant outside the well screen, Eq. 37 can be written as

$$Q^2 = g\, A^2\, (\Delta h_{pz} - \Delta h'_{pz}) \quad (38)$$

Differentiating Eq. 38 with respect to the length L results in

$$\frac{dQ}{dL} = \frac{A^2 g}{2\,Q} \quad \frac{d(\Delta h_{pz})}{dL} = \frac{A\,\sqrt{g}}{2\sqrt{\Delta h_{pz} - \Delta h'_{pz}}}\,\frac{d(\Delta h_{pz})}{dL} \quad (39)$$

However, the increment of discharge dQ into the screen corresponding with an

increment of length dL is

$$dQ = K \frac{\pi D}{e} \Delta h_{pz} \, dL \qquad (40)$$

in which e is the thickness of the gravel envelope.

Combining Eqs. 39 and 40 results in

$$dL = \frac{\sqrt{g}\, e\, D}{8\, K} \frac{d(\Delta h_{pz})}{\Delta h_{pz} \sqrt{\Delta h_{pz} - \Delta h'_{pz}}} \qquad (41)$$

Integration of Eq. 41 yields

$$L = \frac{\sqrt{g}}{4} \frac{e\, D}{4} \frac{1}{\sqrt{\Delta h'_{pz}}} \text{ arc tan} \sqrt{\frac{\Delta h_{pz} - \Delta h'_{pz}}{\Delta h'_{pz}}} \qquad (42)$$

in which the constant of integration equals zero. The value of K for the envelope was found to be 0.00525 m per sec. With **R** equal to 5, it was possible to measure Δh_{pz} in certain sections inside the screen well by using a pitot tube. The results of these measurements (and a comparison of the measured values with those obtained from Eq. 6) are as follows:

Distance between closed end of drain and section, in centimeters	Value of Δh_{pz} from Eq. 42, in meters	Measured value of Δh_{pz}, in meters
0	0.0166	
38	0.0182	0.018
46	0.019	0.019
51	0.0195	0.0195
56	0.0202	0.02
61	0.0215	0.0215
66	0.0229	0.023
71	0.0246	0.025
76	0.0262	0.0265
80	0.0275	0.028
81	0.0278	0.03
82	0.028	0.05

The previously tabulated discrepancies in the value of Δh_{pz} are negligible except for the last sections close to the outlet tube. The discrepancy can be explained by the contraction of flow in the vicinity of the outlet. This contraction increases the velocity and decreases the head inside the tube. Assuming for Δh_{pz} the value 0.0275 m (the last measurement on which the contraction has no influence), it is found from Eq. 40 that $Q = 0.00148$ cu m per sec. The measured discharge was 0.001495 cu m per sec.

When **R** is greater than 5, laboratory tests have shown that the filtration movement is no longer laminar. The discharge is then proportional to $(\Delta h_{pz})^n$ with n having values between 1 and $\frac{1}{2}$. Hence, integration of Eq. 41 becomes impossible.

Jack S. Petersen,[29] Carl Rohwer,[30] and Maurice L. Albertson,[31] Members, ASCE.—Certain interesting ideas worthy of serious consideration have been advanced by Mr. Li. Contrary to Mr. Li's statement, Eqs. 5*a* and 5*b* are correct because they are given simply as the "usual forms." Unfortunately, it was not made clear that, when Eqs. 5*a* and 5*b* are used, the subscripts do not correspond with those in Fig. 1. When applied to the flow approaching and passing through the screen opening and to the jet issuing from the opening, the subscripts must be changed accordingly. Furthermore, the energy which is lost is that of the jet, not that of the longitudinal flow, as implied by Mr. Li. Therefore, Eqs. 5*a* and 5*b* were used in deriving the orifice equation, Eq. 6, which was used to obtain Eq. 14*a*.

Mr. Li states that "* * * one might question the validity of the result for screens surrounded by soil in an aquifer." It is true that the higher the viscosity of the fluid and the lower the permeability of the aquifer, the greater the variation of the piezometric head along the outside of the screen. However, for simplicity in deriving the theoretical equations, this variation was assumed to be of secondary importance. The data (shown in Fig. 11) clearly demonstrate that this assumption was justified.

Within the aquifer at a substantial distance from the screen the flow is undoubtedly laminar, as it must be if Eqs. 19*a* and 19*b* are to be applicable. However, it is possible that for certain conditions the Reynolds number within the gravel envelope is sufficiently high to cause transition flow or even turbulent flow, in which case Eqs. 19*a* and 19*b* are not applicable. If the Reynolds number is greater than 1.0, the flow is no longer laminar; in computing the Reynolds number, D is the grain diameter and V is computed from the bulk area.

Eq. 29, as derived by Mr. Li, is an excellent contribution to the investigation. As in the case of Eq. 16, however, such a derivation is no stronger than its weakest assumption. The final test is its comparison with the data. Therefore, experimental data are necessary to establish the exact influence of each of the parameters in Eq. 29.

In Eq. 32, m is assumed to be independent of the loss coefficient and the screen coefficient, and Eq. 33 further depends on m being equal to unity—all of which are not necessarily true. The writers have evaluated all the data at their disposal (including experiments they have made using a gravel envelope), and in no case is the loss coefficient less than 1.0. Therefore, despite the ingenious derivation of Eq. 33 and the desire of the writers to utilize it, it must be concluded that either the limiting assumptions make it invalid or the range of experimental data is not sufficiently wide to include this special case.

Although the open-bottom wells described by Mr. Collins are used successfully in coping with the problem of sand pumping when suitable clay or rock strata cover the aquifer, there are many areas where these strata are not strong enough to support the overlying material. Where this condition exists other methods of constructing wells have to be adopted. Other types of wells may

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[31] Prof. of Civ. Eng. and Head of Fluid Mechanics Research, Colorado Agri. and Mech. College, Fort Collins, Colo.

sometimes develop large cavities similar to those produced in open-bottom wells; this condition is dangerous because collapse of the roof over the cavity may ruin the well. One reason for the use of gravel-packed wells is that the gravel tends to prevent the formation of these cavities.

It was not the intention of the writers to recommend the construction of any one type of well but, because wells with screens (with or without gravel envelopes) are most common, a study of the hydraulic principles governing the flow of water into such wells seemed desirable.

As to the questions raised by Mr. Collins regarding the tests, it is believed that well drillers should take account of the facts established by the laboratory experiments. The writers, however, are aware that field confirmation of the proposed theory of flow is desirable even though the laboratory tests were in remarkably good agreement with the theory.

The writers are interested in the experiments on the loss of head through screens and gravel packs reported by Messrs. Nazir and Ahmad. The report on the results of loss-of-head tests on screens with larger percentages of openings should be most valuable. Although the results reported in Table 3 show, in general, that the head losses through the screens decrease as the percentage of open area increases, the tests on the screens with larger openings should demonstrate that the losses become constant. The data on the loss of head through the gravel for a constant suction head indicate that the loss decreases as the discharge increases for the larger slit openings. It is difficult to understand why this decrease should occur unless fine material was being washed out of the gravel.

Although the wells in Pakistan have values of $(C\,L)/D$ greater than 30, it does not necessarily follow that all of the screen in the well is effective in decreasing the loss of head. If the writers' analysis is valid, only that part of the screen which will make $(C\,L)/D = 6$ will be utilized (Fig. 17). However, it was not their intention to imply that the loss of head would be the same minimum regardless of the discharge. For each discharge there is a minimum loss which an increase in the length of screen or the percentage of openings will not reduce.

Mr. Peterson's suggestion that a "double effective" length might be developed in a well by properly locating the suction inlet seems reasonable. This phase of the problem was not investigated, but the flow from above the suction inlet—as well as the flow from below the suction inlet—should follow the criterion presented by the writers.

The experiments performed at the University of Ghent, as reported by Mr. Tison, take into account the effect of a special gravel envelope on the head losses through the screen. Although the equipment and the analysis differed from those used by the writers, the head losses follow the pattern reported by them, as shown in Figs. 16, 17, and 18. Unfortunately, the percentage of openings and the coefficient of discharge are unknown for this type of screen. Otherwise it would have been possible to compute the length of screen required to reach the limiting value of $(C\,L)/D = 6$.

The writers are of the opinion that, if the (Δh_{pz})-values for distances of 82 cm (32.3 in.) and 86 cm (33.8 in.) between the closed end of the drain and the

section had been measured, still larger head losses would have been observed. It is also believed that the observed losses reported by Mr. Tison may be more nearly correct than the computed losses.

Problems involving many variables can be approached in many ways; different investigators may derive formulas of different form for the solution of the same problem by using a different approach. Eq. 34 reported by Mr. de Lara, provides for the thickness and permeability of the gravel envelope whereas Eq. 15 takes account of the gravel envelope in so far as it affects the value of C by changing the orifice coefficient and the effective open area of the screen. As shown in Figs. 14 and 15, the loss through the screen is independent of the gravel envelope except when the percentage of open area in the screen is inadequate and the size of the gravel is small.

An important relationship based on Fig. 13 is shown by Mr. de Lara in Table 4 and Fig. 23. Fig. 23 shows that the curves for the different slot openings in Fig. 13 can be combined into a single curve by substituting the ratio d/s for the gravel size. However, when d/s is less than 1, the gravel particles would not be held back effectively by the screen, and an unstable condition would result. For this reason the part of the curve for d/s less than 1 should either be eliminated from Fig. 23 or be shown as a broken line. No doubt the screen coefficients for the type C screens could also be plotted theoretically on a single curve because of the geometrical similitude of the slots in these screens. However, these screens have a relatively large percentage of open area, and as a result most of the tests were in the range in which the head losses were a minimum—that is, $(C\,L)/D$ greater than 6. Consequently, sufficient data are not available for the analysis because the effective length of the screen is unknown.

The writers recognize that the conclusions stated in the paper require confirmation by field tests. Their primary purpose was to call attention to the peculiarities of flow into wells that had not been previously reported.

As noted by Mr. Rorabaugh, the data are based on tests of screens that do not penetrate the aquifer completely. The original plan for the tests included the installation of flexible disks in the gravel at the top and bottom of the perforated area of the screens so that all the water would approach the screen radially and horizontally. This plan was not satisfactory because the disks moved as the gravel settled. The disks also produced a plane of different permeability at the top and bottom of the screen. For these reasons the disks were not used.

It should be noted that complete penetration of the screen would not have eliminated the converging flow lines because the tests showed that most of the water entered the screen at the discharging end. This condition tends to concentrate the flow lines at the discharge end. The same effect is shown by the tests reported by Mr. Tison and by Mr. Li in Fig. 19.

The head loss through the screen and the gravel pack is a small fraction of the total drawdown because most of the head loss occurs in the aquifer. The loss in the aquifer depends on the discharge and the characteristics of the aquifer and not on the screen or the gravel pack.

In Mr. Rorabaugh's analysis of the drawdown in a well in an artesian aquifer under conditions of full and partial penetration it is shown that the drawdown decreases as the depth of penetration increases. No doubt this is true under certain conditions, but the writers believe that it may not be true if the length of screen required to make the value of $(C\,L)/D$ equal to 6 is less than the depth of penetration. So far as is known, these two conditions have never been checked by actual tests. Such a test is possible and would certainly yield useful results.

The writers appreciate the interest shown by all those who have contributed discussions. It is hoped that this work will lead others to investigate the problem by conducting field tests wherever conditions are favorable.

Corrections for Transactions.—Eq. 8*b* should be changed to $-A^2\,g\,dh_{pz} = d(Q^2)$. Eq. 9*a* should be changed to Δh_{pz} = a constant $- h_{pz}$. Eq. 9*b* should be changed to $d(\Delta h_{pz}) = -dh_{pz}$. In Eqs. 14*a* and 14*b* the radical signs in the denominators of the right-hand members should be extended to cover the terms, $\Delta h'_{pz}$. All parentheses should be removed from Eqs. 15 and 16.

Discussion of
"HYDRAULIC MODEL STUDIES OF MARTIN DAM DRAFT TUBES"

by Carl E. Kindsvater and R. R. Randolph, Jr.
(Proc. Paper 382)

RICHARD S. WOODRUFF,[8] A. M. ASCE.—Pyramids were installed in the

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three original draft tubes in September, October, and November, 1952, and were of the dimensions recommended on the basis of the authors' research (Fig. 11). When the draft tubes of Units 2 and 3 were unwatered, large irregular pieces of concrete were found immediately inside the draft-tube exit. The shapes and sizes of these pieces as well as the nature of the broken material indicated that they were from the original half cones. No sections of the ¼-in. steel plate from the surface of the original cones were found in these two tubes. The draft-tube exit of Unit 1 was clear because approximately 75% of the original cone concrete in this tube was still in place. In this tube it was an easy matter to chip out the remains of the old cone because the concrete was of inferior quality and most of the steel plate had washed away. The steel shells of the original cones were filled through a small opening at the top; this was done despite a considerable handicap caused by leakage water from the penstock and the threat of flood-water conditions in the tailrace.

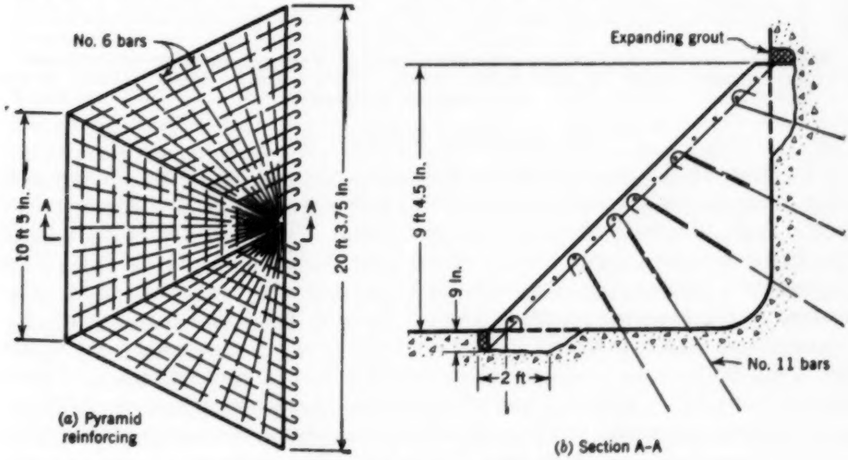

FIG. 12.—DIAGRAM SHOWING REINFORCING BARS

The new pyramids were installed by segregating (with sandbags) and drying the part of the draft tube involved. The original concrete surface was thoroughly cleaned of scum and was carefully chipped to clean, sound material after which No. 11 reinforcing rods were grouted at least 4 ft deep into the old concrete (Fig. 12). Wooden forms were lined with an absorptive form liner ⅜ in. thick in order to produce a smooth, hard, dense surface on the finished pyramid. Forms were not lowered into the draft tube until a few hours before pouring began because of the wet and humid condition. After the forms were assembled, several large light bulbs were installed inside them in order to help keep the absorptive liner dry. Concrete for the pyramids was a seven-sacks-per-cubic-yard design with an air-entraining agent; slump of from 2 in. to 4 in.

was specified. The average 28-day strength of concrete test cylinders was 3,800 lb per sq in. A man was inside the form during the pouring process to work the concrete along the face of the forms with a spade as a precaution against honeycombing. A vibrator was used during the pouring, but special care was taken to prevent it from contacting the anchor dowels as the concrete in the bottom of the form had reached its initial set before the pyramid was "topped out." The forms were removed 36 hr after concrete was poured, and the draft tube was flooded a day later. Each pyramid was allowed to cure for 14 days with the unit shut down. From 14 days to 21 days after pouring, the unit was restricted to generation at best gate in order to keep turbulence in the draft tube at a minimum. After 21 days, the unit was released for unrestricted operation.

Excellent results were obtained in producing a smooth surface on the pyramid and in using expanding grout around the periphery. The pyramids for all three units were installed at a total cost of approximately $41,000.

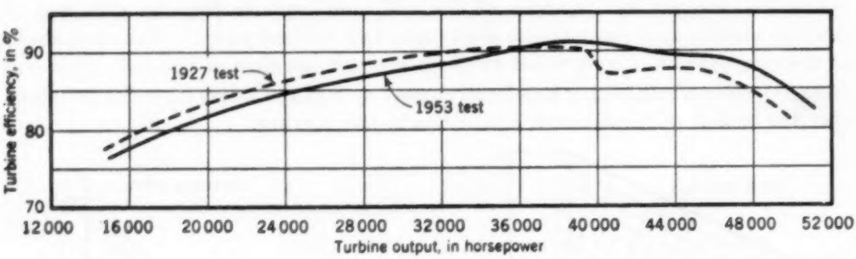

Fig. 13.—Turbine Efficiency, Unit 1

In June, 1953, a second Gibson test was performed on Unit 1 for the purpose of comparing the performance of this unit after the pyramid was installed in the draft tube with the 1927 acceptance test. This test disclosed that the maximum turbine output was increased from 50,500 hp to 51,330 hp. This represents a gain of approximately 605 kw in generating capacity, which conformed to the expected results. The efficiency at maximum capacity was increased from 79.5% to 82.2%. Best gate operation was increased from 36,400 hp to 39,000 hp with a corresponding increase in maximum efficiency of from 90.3% to 91.1%. Although the efficiency decreased approximately 0.7% for gate openings producing less than 36,000 hp, its average increase was more than 1% for those producing more than 36,000 hp. Both tests were based on a net head of 145 ft. The increased efficiency is in the range of operation where it will be most beneficial. The "kink," or abnormal drop in efficiency that occurred near 40,000 hp, without the modification, was definitely smoothed out. This is shown by a comparison of the two Gibson-test curves (Fig. 13).

A comparison of the operating characteristics of Units 1, 2, and 3 at Martin Dam results in the following conclusions:

1. An increase in generating capacity and efficiency was obtained by the installation of the half pyramid in the heel of the draft tubes (Fig. 10(*b*)), thus again indicating the value of hydraulic model studies.

2. The installation was economically justifiable because the cost was offset by the value of the increased energy and capacity in approximately one year.

Carl E. Kindsvater[9] and Richard R. Randolph, Jr.,[10] Members, ASCE.

[9] Prof. of Civ. Eng., Georgia Inst. of Technology, Atlanta, Ga.
[10] Hydr. Engr., Southern Services, Inc., Birmingham, Ala.

—Mr. Woodruff's discussion is a necessary complement to the writers' description of the events leading to the latest attempt to remedy the undesirable characteristics of the original draft tubes in the Martin Dam power plant. The details of design and the methods used in constructing the remedial structure are of particular interest because they were devised in the light of the unfortunate experiences associated with the modifications made in 1928. The disintegration of the original remedial structures, revealed when the tubes were unwatered, is sufficient evidence of the necessity for the meticulous care with which the latest construction was accomplished.

Mr. Woodruff reports that concrete remnants of the original heel cones were of inferior quality. The poor quality of the concrete used was doubtlessly indirectly related to the difficulty of placing the backfill under the outer steel shell. The combination of an infirm backfill and an insufficient anchorage apparently accounted for the failure of the steel armor under the unsteady dynamic loads resulting from turbine discharge. The simplicity of the new pyramid-shaped structures greatly facilitated the construction described by Mr. Woodruff. It is believed, furthermore, that the omission of a steel armor and the concentration on the production of superior-quality concrete which he describes shows a proper recognition of the necessity for fitting materials and construction methods to the conditions of the job.

The writers were particularly pleased that comparative efficiency tests were made on Unit 1 before and after the remedial construction was completed. Prototype verification of conclusions based on hydraulic model studies is admittedly viewed with considerable relief by the researcher. It is true that the basic principles of similitude are as indisputable as the most fundamental principles of engineering mechanics. Unfortunately, however, the requirements for complete similitude are not always possible to fulfil in the laboratory. It follows that a considerable amount of judgment, based on experience as well as theory, is an important component of every model study. The omniscient Leonardo is quoted as a final warning: "Experiment never errs; only your judgment errs in anticipating results which experience does not confirm."

In connection with Mr. Woodruff's discussion of the prototype tests, it is pertinent to emphasize the impracticality of a direct comparison of the results of the turbine efficiency tests with the results of the hydraulic model tests reported in the paper. Sufficient evidence is believed to be available to substantiate the use of "diffuser efficiency" as an indication of the relative worth of different draft tubes. The writers do not mean to imply, however, that the draft-tube component of the power efficiency of a turbine can be determined from Eq. 5. For this investigation specifically there is little reason to believe that general draft-tube characteristics need be determined from a scale model of the complete machine. Indeed, it was the simplicity of the laboratory apparatus used by the writers that was unique in comparison with previous studies.

It was observed following the derivation of Eq. 5 that η is an adequate

measure of the energy efficiency of a diffuser only when the area ratio (A_3/A_1) approaches zero. It should also be acknowledged that the writers' interpretation of Eq. 5 is inadequate as a means of determining the true energy efficiency of any diffuser unless the flow at the end sections is both uniform and axial. It follows that, because the effect of whirl was ignored in evaluating Eq. 5 from the laboratory data, an error of indeterminate magnitude is inherent in certain of the computed efficiency values. This does not of, course, invalidate the procedure used to determine the relative merits of the various draft-tube modifications.

In the derivation of Eq. 5, it was implied that the flow was both uniform and axial. For this condition, the average piezometric head at both Section 1 and Section 3 is represented by the piezometric head at the boundaries. This fact is important, because only at the boundaries are the piezometric heads readily measured. With whirl at Section 1, however, the piezometric head actually varies from a minimum at the center to a maximum at the wall. The magnitude of this variation depends on the magnitude and distribution of the velocity in the cross section. Unfortunately, neither the velocity pattern nor the piezometric-head distribution can be measured by ordinary means. It follows, therefore, that the use of wall piezometers to evaluate h_1 results in an increasing overevaluation as the whirl increases. At Section 3, of course, the residual whirl is usually so small that it has negligible influence on the normal, hydrostatic pressure variation in the section.

The second complication resulting from whirl at Section 1 concerns the denominator of Eq. 5. Thus, when the flow contains a whirl component, V_1 is correctly defined as the vector sum of the axial velocity and the tangential velocity in the section. For a given discharge, therefore, the average kinetic energy at Section 1 increases with increasing whirl. The exact magnitude of this influence is also indeterminate. It was expedient and consistent with the objectives of the investigation, therefore, to evaluate V_1 as the average axial velocity, Q/A_1.

Discussion of
"TRANQUIL FLOW THROUGH OPEN-CHANNEL CONSTRICTIONS"
by Carl E. Kindsvater and Rolland W. Carter
(Proc. Paper 467)

EMMETT M. LAURSEN,[11] A.M. ASCE, and ARTHUR TOCH,[12] J.M. ASCE.—

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Within the limitations of the range of geometries used in the model investigations, the coefficients presented by the authors should aid greatly in the indirect determination of streamflows. The scatter of ± 5% in Fig. 10 indicates a higher degree of accuracy for the discharge coefficient than is likely to be obtained for Δh and m in the field. Because the limitations recognized by the originators of such solutions as this are too often misunderstood or ignored, two major limitations on the applicability of this method of determining stream discharges should be mentioned.

If the stream banks between zones 2 and 3 are covered with brush and trees, the water-surface elevation on the downstream side of the constriction may not be equal to that in the constriction. Depending on the density of the vegetal screen, the elevation of the water surface in zone 3 rather may more nearly approximate that elevation in zone 4. Although laboratory experiments with screens of known solidity ratio could determine this effect, in practice the difficulty of determining the density of a brush and tree cover would still remain. The companion work[13] to the paper might be used to obtain Δh in

[13] "Backwater Effects of Open Channel Constrictions," by Hubert J. Tracy and Rolland W. Carter, *Transactions* ASCE, Vol. 120, 1955

this case, but in using the curves therein, it is suggested that m be determined from the downstream discharge ratio. Because the slopes of the main channel and the flood plain would not be the same, the conveyance ratio should not be used.

If the bed of the stream is erodible, scour in the constriction may occur during a flood and the scour hole may be completely or partly refilled by the time the elevation of the bed of the stream is determined. Considerable error could thus, of course, be caused in the estimation of the depth of flow. An approximate indication of the order of magnitude of this effect has been made[14]

[14] Discussion by Emmett M. Laursen and Arthur Toch of "River-Bed Scour During Floods," by E. W. Lane and W. M. Borland, *ibid.*, Vol. 119, 1954, p. 1084.

by the writers.

The writers wish to express their admiration for the careful experimental work which forms the basis of this paper. It is agreed that the analysis, admittedly empirical, is the only one feasible; much more information is needed about the fundamentals of turbulent flow before a more rational analysis is practicable.

FRED W. BLAISDELL,[15] M. ASCE.—When the results of a comprehensive

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series of generalized tests are summarized in the few curves presented by Messrs. Kindsvater and Carter, it is difficult to realize the tremendous effort and large number of tests that are required to obtain these results. The writer and Charles A. Donnelly completed (in 1950) a generalized study of the capacity of the box inlet drop spillway;[16] the methods of analysis and presenta-

[16] "Hydraulic Design of the Box Inlet Drop Spillway," by Fred W. Blaisdell and Charles A. Donnelly, *Technical Paper No. 8-B*, St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn., January, 1951.

tion closely paralleled those used by the authors. Because of this experience, the writer can fully appreciate what is meant by the statement (under the heading, "Simple Constrictions in Rectangular Open Channels: Experimental Verification of the Analysis") that "After much study, a procedure was adopted * * *." The benefit is well worth the effort for, in presenting their results, the authors have done much of the work required for the practical application of the results of their research.

As noted by Messrs. Kindsvater and Carter, the discovery that variations in the geometrical shape of the channel cross section can be expressed in terms of conveyances is certainly an interesting and valuable contribution. It is to be regretted that values of m less than 0.20 were not included in the test schedule. It would have involved only a little more work and would have eliminated questions that always arise when curves are extrapolated beyond the range of the tests. It may be that these low values of m are assumed to be outside the range of practical application. However, there may arise practical situations in which values of m will be less than 0.20. It has been the writer's practice to extend the range of laboratory tests well beyond the "practical" limits set by those experienced in the actual design of structures. Even so, subsequent practical applications outside the extended range of test variables have been frequent.

Tests at values of m less than 0.20 would have been particularly helpful in defining the effects of angularity, corner rounding, and guide walls. In the case of the adjustment curves for angularity, a large part of the curvature is at values of m less than 0.20. Although the curves are shown to be solid in Fig. 8, one must assume (based on the table in Fig. 10) that they must have been drawn without the benefit of experimentally determined points.

The comparison of the adjusted and observed values of C presented in Fig. 10 provides a crucial test of the analytical methods. The agreement shown is excellent. Although it is undoubtedly possible to obtain less spread in the data for individual curves, greater spread must be expected as adjustments are added. Overcompensating benefits include an expanded range of application and an increased reliability resulting from the greater volume of data required to define the coefficient and adjustment curves.

PIN-NAM LIN.[17]—The discharge coefficient has been virtually defined by

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Messrs. Kindsvater and Carter in two ways. It is defined in one case by the first dimensionless group in Eq. 6*b* and in the other by Eq. 10. Because h_f and $\alpha_1 V^2_1/2\,g$ in Eq. 10 have opposite signs it is possible that in many cases the two values of C are approximately the same. In general, the two definitions will yield different values of C; the case of channels having considerable tree growth and brush, cited by the authors, is an example. The apparent discrepancy, however, appears to be the result of an error in presentation because, if the first definition were to be considered, Eq. 6*a* should have included the coefficient α_1 as well as a variable to indicate the roughness of the channel. For a given boundary geometry at a given locality, Δh will vary not only with Q and y_2 but also with the channel roughness and the velocity distribution upstream of the constriction. When C is defined by Eq. 10, it is

reasonable that C will be expressed by Eq. 12 or Eq. 17.

Flow through channel constrictions at high values of m is accompanied by considerable surface curvature. As a result of the surface curvature, the acceleration in a vertical plane may be considerable. Analysis of a flow that is no longer "gradually varied" was first conducted by Boussinesq.[18,19]

[18] "Steady Flow in Open Channels: The Problem of Boussinesq," by C. Jaeger, *Journal*, Inst. of C. E., London, England, February, 1948, pp. 339–349.

[19] "Fluid Mechanics for Hydraulic Engineers," by H. Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, pp. 301–307.

Because the pressure distribution is no longer hydrostatic in the section where the surface curvature is pronounced, coefficients should also be introduced to account for the effect of pressure distribution on the static-head terms in the energy and momentum equations. Because the surface curvature at section 2 may be considerable, it would be more appropriate to write Eq. 7 as

$$\alpha_1 \frac{V^2_1}{2\,g} + h_1 = \alpha_2 \frac{V^2_2}{2\,g} + \beta\, y_2 + h_e + h_f \qquad (21)$$

in which

$$\beta = \frac{1}{Q\, y_2} \int\!\!\int_{A_2} \left(\frac{p}{w} + z\right) dQ \qquad (22)$$

In tranquil flow, the static head above the bottom of the channel is usually the major part of the specific head so that—depending on the surface curvature—β may be an important coefficient. In fact, because the flow at section 2 is markedly convergent the velocity distribution in this section should be approximately uniform, and thus α_2 probably does not differ greatly from unity; that is, α_2 is often of secondary importance. From Eq. 21, one would be inclined to include the coefficient β in Eq. 9. It is believed that this step is probably not necessary in the present case, in which the main objective is to compute the discharge on the basis of experimental coefficients of discharge. One may consider Eq. 10 or Eq. 13 as a formula defining C, which is expressed by Eq. 12 or Eq. 17; the effect of surface curvature is included in Eqs. 12 and 17.

If, however, Eqs. 9 and 10 are regarded as rational deductions from the equations of energy and continuity (that is, Eqs. 7 and 8), the coefficient β should be included in the energy equation as indicated by Eq. 21. Boussinesq assumed[20] that the curvature in a vertical section varied linearly from the

[20] "Technische Hydraulik," by C. Jaeger, Birkhäuser, Basel, Switzerland, 1949, pp. 116–122.

surface to the bed and obtained, for the case of uniform velocity distribution in a vertical direction, the formula:

$$\frac{p}{w} = y - z + \frac{V^2}{g\, y} \frac{d^2 y}{dx^2} \left[y\,(y - z) - \frac{(y - z)^2}{2} \right] \qquad (23)$$

Assuming further that the velocity distribution is uniform in section 2, there results

$$\beta = \frac{1}{Q\, y_2} \int\!\!\int_{A_2} \left(\frac{p}{w} + z\right) dQ$$

$$= \frac{C_c\, b}{Q\, y_2} \int_0^{y_2} \left(\frac{p}{w} - z\right) V_2\, dz$$

$$= \frac{1}{y^2_2}\int_0^{y_2}\left\{y_2 - \frac{V^2_2}{g\,y_2}\left(\frac{d^2y}{dx^2}\right)_2\left[y_2\,(y_2 - z) - \frac{(y_2 - z)^2}{2}\right]\right\} dz$$

$$= 1 + \frac{1}{3}\,\frac{V^2_2}{g\,y_2}\left(y\,\frac{d^2y}{dx^2}\right)_2 \qquad (24)$$

In general, C. Fawer[21] showed that, by assuming the curvature at any level

[21] "Étude de quelques écoulements permanents à filets courbes," by C. Fawer, La Concorde, Lausanne, Switzerland, 1937.

as given by $(z/y)^n\, d^2y/dx^2$, Eq. 24 could be extended to yield

$$\beta = 1 + \frac{1}{n+1}\,\frac{V^2_2}{g\,y_2}\left(y\,\frac{d^2y}{dx^2}\right)_2 \qquad (25)$$

Naturally, both Eqs. 24 and 25 are far from being exact. These expressions are, nevertheless, at least qualitatively correct. Because the surface at section 2 is concave d^2y/dx^2 is positive, and therefore $\beta > 1$. This result may be anticipated if it is noted that the centrifugal effect of flow at section 2 is to increase the local pressure beyond that predicted by the hydrostatic pressure distribution. For a given value of m, different cases of flow through a given type of constriction should be geometrically similar. As a result, corresponding to a given value of m, the dimensionless parameter $y(d^2y/dx^2)$ should be approximately constant. On the basis of Eqs. 24 and 25, several qualitative observations can thus be made concerning β for flow through a given type of constriction: (1) Even if m is kept constant, β still increases with the Froude number. (2) Regardless of the magnitude of m, β approaches unity as the Froude number approaches zero. (3) The term β increases with m. These observations indicate clearly that the magnitude of β depends not only on the surface curvature of flow but also on the Froude number.

In order to gain some idea regarding the influence of the coefficient β, the possible effect of β on the ratio $(y_1 - y_4)/\Delta h$ will be cited briefly. Because the value of β at section 2 is greater than unity, the total energy per unit volume of water at section 2 is greater than that indicated by $(y_2 + \alpha_2\, V^2_2/2\,g)$. The value of Δh computed by the energy equation without taking β into account will therefore be less than the actual value of Δh. The effect of β is thus to increase the value of Δh. However, the quantity $(y_1 - y_4)$, being governed essentially by conditions in sections 1 and 4, may be considered independent of β. Therefore, the resultant effect of β is to reduce the ratio $(y_1 - y_4)/\Delta h$. In the light of the three observations previously made regarding β, the following remarks concerning $(y_1 - y_4)/\Delta h$ can be made:

a. For a given value of m, $(y_1 - y_4)/\Delta h$ decreases as the Froude number increases.

b. For a given value of the Froude number, the decrease in $(y_1 - y_4)/\Delta h$ is greater as m increases.

Both of these trends appear to be supported by experimental data.[22] Consequently, within the range of data presented by Messrs. Kindsvater and Carter, it seems certain that a more rational analysis of the problem should include a consideration of β.

[22] "Backwater Effects of Open Channel Constriction," by Hubert J. Tracy and Rolland W. Carter, *Transactions*, ASCE, Vol. 120, 1955, Figs. 6–8.

Carl F. Izzard,[23] A.M. ASCE.—The placing of a constriction in an open channel inevitably results in an increase in the upstream stage over the stage which existed for the same discharge in the unconstricted channel. In Fig. 11 there is illustrated the change in profile from flow at a uniform depth y_n to a typical backwater flow. Also shown are the lines of total head obtained by plotting the velocity head above the free water surface, assuming that one can estimate the kinetic energy in the flow. The depth y_1 is the increased depth which Messrs. Kindsvater and Carter have measured at a section a distance b upstream from the upper edge of a constriction whose width normal to the direction of flow is b. To generalize the analysis, the channel is shown to have a slope S_0 which actually was zero in the authors' experiments.

[23] Chf., Hydr. Research Branch, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

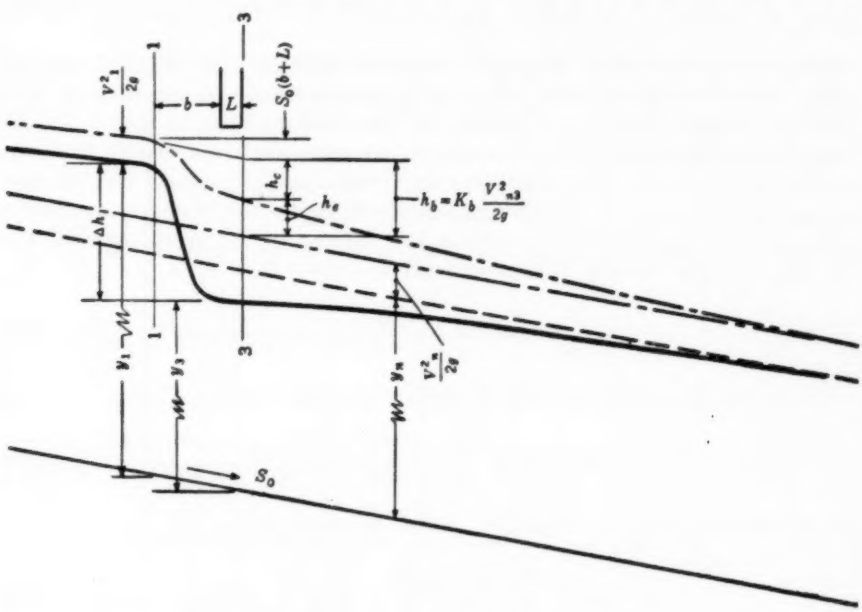

Fig. 11.—Definition Sketch of Backwater

Using the nomenclature adopted by the authors, section 3 is taken at the downstream edge of the constriction. The drop in the total-head line from section 1 to section 3 can be considered to be equal to the normal resistance loss plus the loss h_c caused by the contraction of flow and is probably small compared to the total loss. This flow, however, has to expand and in so doing additional energy is released that is represented by the difference h_e between the total head at section 3 and the total head with uniform flow. The sum

$$h_c + h_e = h_b \qquad (26)$$

thus represents the total energy loss in excess of the normal friction loss between section 1 and a section downstream where the flow is again at normal depth.

From Fig. 11 it is apparent that h_b is also equal to the change in total head at section 1.

The energy equation between section 1 and section 3 then becomes

$$S_0\,(b + L) + y_1 + \frac{V^2_1}{2\,g} = y_n \frac{V^2_n}{2\,g} + h_b + S_0\,(b + L) \quad (27)$$

Where a cross section is irregular, a suitable value for the weighted velocity head should be substituted. Because h_b results from a contraction of flow one can assume that it is a function of the velocity head at section 3; the experiments, however, have shown that the water surface is not level transversely at section 3. To avoid this complication one can assume that the area at section 3 is taken at the normal depth, relying on the head-loss coefficient determined empirically to correct for the difference. Using the symbol V_{n3} to designate the velocity at normal depth in section 3,

$$h_b = K_b \frac{V^2_{n3}}{2\,g} \quad (28)$$

Substituting this value into Eq. 27 and transposing terms,

$$y_1 - y_n = \frac{V^2_n}{2\,g} - \frac{V^2_1}{2\,g} + K_b \frac{V^2_{n3}}{2\,g} \quad (29)$$

From the continuity equation, Eq. 29 can be expressed as

$$y_1 - y_n = \frac{V^2_{n3}}{2\,g}\left[\left(\frac{A_{n3}}{A_n}\right)^2 - \left(\frac{A_{n3}}{A_1}\right)^2 + K_b\right] \quad (30)$$

from which

$$K_b = \frac{y_1 - y_n}{\dfrac{V^2_{n3}}{2\,g}} - \left(\frac{A_{n3}}{A_n}\right)^2\left[1 - \left(\frac{A_n}{A_1}\right)^2\right] \quad (31)$$

If A_n/A_1 approaches unity as it will for relatively small quantities of backwater, an approximate equation for K_b is

$$K_b = \frac{y_1 - y_n}{\dfrac{V^2_{n3}}{2\,g}} \quad (32)$$

Backwater can then be estimated as

$$y_1 - y_n = K_b \frac{V^2_{n3}}{2\,g} \quad (33)$$

provided values of K_b for a given type of constriction are known. After a trial value of y_1 is determined from Eq. 33, a more accurate value can then be obtained using Eq. 30 because a close approximation of A_1 will then be known.

Experiments have been described[13] from which it has been possible to test the applicability of Eqs. 29 to 33. In those tests steady flow was first established with no constriction, and an equivalent of normal depth was recorded.

The flume used had a level floor, but the change in depth between section 1 and section 3 was usually very small in relation to the change in depth between the same sections with the constriction in place. The most extensive data were for a vertical constriction with a symmetrically placed rectangular opening. In Fig. 12 it is shown that K_b varies systematically with the contraction ratio m and to a lesser extent with the parameter $V^2_{n3}/(2\,g\,y_n)$. In Fig. 12 only

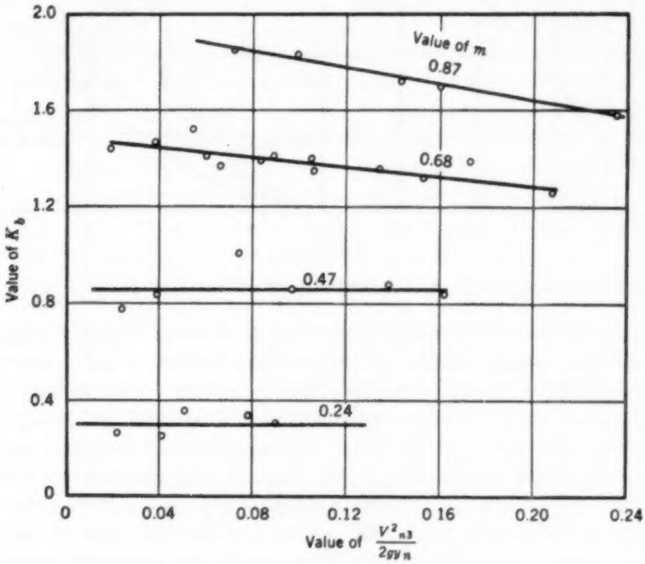

FIG. 12.—BACKWATER COEFFICIENT AS A FUNCTION OF VELOCITY HEAD RELATIVE TO NORMAL DEPTH ($n = 0.012$, $L = 0.02$ ft)

tests made with $L = 0.02$ ft and $n = 0.012$ are shown. The correlation is good except for a few tests, in which the surface profile for flow with no constriction appeared to be irregular. A more significant plot of the data is shown in Fig. 13(a) which is derived from the curves drawn in Fig. 12. From Fig. 13(a) it is indicated clearly that K_b is primarily a function of m, the variation with the velocity-head parameter being of minor importance; similar and slightly lower curves could be drawn for other values of L. Roughness of channel appears to affect K_b slightly, but this difference may be the result of the increased head required to obtain flow on a level floor.

A few runs were made with other types of constrictions. The computed values of K_b for two types are shown in Fig. 13(b) in comparison with the curve for the rectangular opening. The 45° wing wall had a constant dimension $w = 0.25$ ft so that w/b was variable with the value of b and therefore with m; this accounts for the tendency of the curves for the wing-wall type to approach the curve for a rectangular opening when w is relatively small, and to approach the curves for the rounded embankment for the case in which w is relatively large. The radius at the toe of the 1 on 1 rounded embankment was not constant, this radius varying from one third to one half of the distance across the constriction between the toe of the slope on each side.

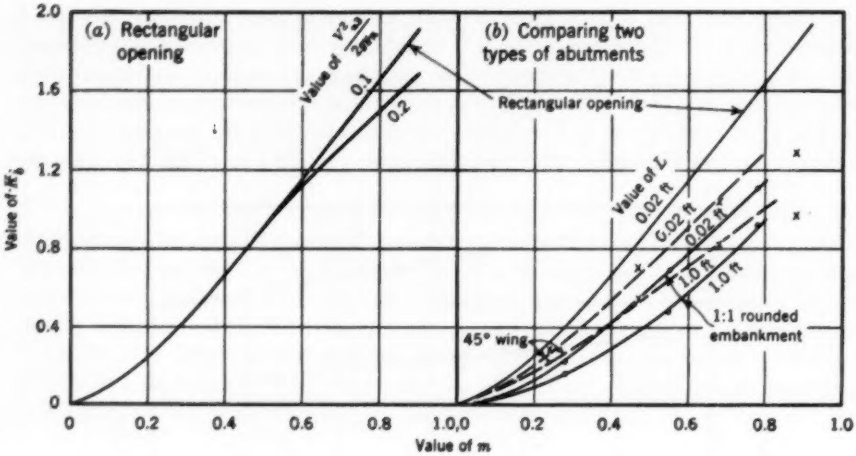

FIG. 13.—BACKWATER COEFFICIENT AS A FUNCTION OF CONTRACTION RATIO (n = 0.012)

These analyses, based on meager experimental data, strongly suggest the possibility—using a tilting flume—of determining values of K_b that would be applicable to standard types of bridges and that would be independent of channel roughness. If so, backwater could be computed by using Eq. 33. By applying this method the contracted depth y_3 at the downstream side of the abutment would not be given, but this value is not important to the bridge designer. Actually $y_n - y_3$ is a function of $V^2{}_{n3}/2\,g$ similar to Eq. 33.

The foregoing discussion has been directed at the problem of estimating backwater caused by constricting a channel carrying a known flow at normal depth, which is the problem of the highway engineer. Messrs. Kindsvater and Carter, however, were confronted with the reverse problem of estimating discharge from measurement of water levels in the vicinity of a channel constriction. Because the depth y_3 is sensitive to small changes in geometry, it was necessary to devise a system which would reflect these changes; this they have done with remarkable ingenuity.

Performing the tests in a channel with a level floor greatly simplified the experimental procedure but left some question as to the applicability of the discharge coefficient to flow in a sloping channel, particularly when h_f is large in comparison to Δh. The basic equation defining C (Eq. 13) includes a term for head loss in the approach reach computed by the geometric mean of the energy slopes at sections 1 and 3 and the energy slope through the contracted reach. The evaluation of C would have been simpler if h_f had been computed solely for the friction loss in the unobstructed channel over the length $b + L$. All the energy losses attributable to the geometry of the constriction would therefore be included in the discharge coefficient. Thus, one would avoid the question as to whether or not the friction loss is correctly evaluated by the geometric mean of the energy slopes in the case in which the transition is not gradual but abrupt.

For computing discharge for an actual bridge, the normal friction loss could be checked against the energy slope in an unobstructed reach of the same cross section and should be approximately equal to the average bed slope. Thus,

the uncertainties involved in estimating Manning's n—required to compute the friction loss—would be avoided. It would still be necessary to use n in estimating the contraction ratio but, because n appears in both numerator and denominator, the effect of error would be minimized. Consequently, the evaluation of C in Eq. 13, using h_f as the normal friction loss, would simplify computation in the field. Further simplification could be attained by defining m as K_q/K_Q instead of $1 - K_q/K_Q$. The obvious term for m would then be "conveyance ratio."

The problem of estimating discharge from field measurements requires that the elevations h_1 and h_3 must be measured at certain points to determine Δh. In estimating backwater $y_1 - y_n$, it is not necessary to compute y_3 if K_b in Eq. 33 is known from experimental data. The computation is simple and gives results which appear to be as consistent, if not more so, than those obtained by computing Δh from Eq. 13 and then applying corrections[13] to get $y_1 - y_n$.

In a discussion[24] the writer demonstrated that y_1/y_n was a function of

[24] Discussion by Carl F. Izzard of "Backwater Effects of Open Channel Constrictions," by Hubert J. Tracy and Rolland W. Carter, *Transactions*, ASCE, Vol. 120, 1955

$V^2_{n3}/(2\,g\,\bar{y}_n)$ and the parameter m for a given type of constriction. By subtracting unity from the ordinate scale and multiplying both scales by y_n, these quantities take the form of Eq. 33 in which K_b becomes the slope of a straight line approximating the curved lines. The reason for the consistency of the data in relation to the velocity-head parameter based on normal depth in the contracted opening is that the total head loss caused by the contraction h_b (as defined in Fig. 11) is referred to total energy at normal depth.

The use of Eq. 33 together with charts similar to Fig. 13(b) is believed preferable to the use of another chart[25] because Eq. 33 gives the increment in

[25] *Ibid.*, Fig. 9.

depth and not a depth relative to a normal depth which might prove difficult to define for irregular channels. More experimental data are needed, preferably from a sloping flume, to define K_b for standard types of bridges

Chesley J. Posey,[26] M. ASCE.—The work of Messrs. Kindsvater and

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Carter should do much to clarify the thinking of engineers on a problem that has often proved puzzling. The tests reported not only provide useful information but also point the way for future experimentation that will be even more valuable.

The authors properly emphasize the importance of accounting for the contraction of the jet. They omitted one factor, however, that can have a considerable effect on the contraction—friction along the upstream face of the obstruction; the corresponding orifice effect is recorded in standard text books on hydraulics.[27] Messrs. Kindsvater and Carter should have included some

[27] "Hydraulics," by H. W. King and C. O. Wisler, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1933, p. 101.

tests with variable roughness in this location inasmuch as they did vary the bed roughnesses.

In their future tests, the authors should give special attention to the contraction to be expected under various conditions. The right of way along

the fill section approaching a bridge is apt to be kept cleared whereas the flood-plain bottom beyond the right-of-way lines is often overgrown with underbrush and trees. Hence, the flow parallel to the fill approaching the bridge will attain a higher velocity than it would under the flow-net assumption of equal (or nonexistent) friction throughout the approach area. Entering the main flow transversely just above the bridge opening, this current of high velocity will cause more contraction than would otherwise exist. Not only is this condition almost universal but it is often accentuated by the presence of borrow pits or large ditches along the side of the fill.

The designation of some formulas as "rational" and others as "empirical" seems unfortunate. Because all formulas representing physical phenomena are empirical, what is probably meant is that some have more claim to rationality than others.[28] The rationality of the Weisbach formula is not at all appealing but, because it includes two terms with different exponents (and hence is similar to a power series), it can be fitted more closely to the experimental data than can other formulas that have a more logical basis but only one term.[5] A close examination of the Nagler formula[9] shows that it has at least as good a claim to "rationality" as has, for example, the application of the flow net to open-channel problems. The authors' introduction of the concept of contraction in the study of open-channel flow is not new[8,29] but, when correlated with systematic experimental studies, it seems to bring order out of chaos. One wonders whether Messrs. Kindsvater and Carter could predict the experimental results obtained[5] by Mr. Lane by an a priori evaluation. Perhaps their suggestion that the degree of jet contraction should be proportional to the ratio *m* can be provided with "adjustments" to take into account the existence and relative elevations of side borrow pits, ditches, or cleared right of ways.

[28] "The Science of Mechanics," by Ernst Mach, Open Court Publishing Co., Chicago, Ill., 5th Ed., 1942.

[29] "An Experimental Study of the Flow of Water through Transitions in Rectangular Open Channels," by George B. Lyon, thesis presented to the State University of Iowa, Iowa City, Iowa, in February, 1942, in partial fulfilment of the requirements for the degree of Master of Science.

Carl E. Kindsvater,[30] M. ASCE, and Rolland W. Carter,[31] A. M. ASCE.—The subject of this paper was the first phase of a comprehensive investigation of the flow of water through bridge openings. Since the delivery of the paper at New Orleans, La., in 1952, two additional reports on related research have been published. In the first of these[32] the study of discharge characteristics is extended to include additional boundary conditions; the second[13] deals with the flow pattern downstream from the constriction. As the basis for these and other investigations still in progress, the writers' paper is appropriately complemented by the excellent discussions submitted by Messrs. Laursen and Toch, Blaisdell, Lin, Izzard, and Posey.

Messrs. Laursen and Toch refer to certain limitations on the applicability of the writers' solution. They observe correctly that the relationship between the level of the water in zone 3 and that in zone 2 depends somewhat on the density and distribution of the vegetal cover on the banks of the stream. It

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[32] "Computation of Peak Discharge at Contractions," by Carl E. Kindsvater, Rolland W. Carter, and Hubert J. Tracy, *Circular No. 284*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1953.

should be emphasized, however, that the experimental coefficients are based on measurements made at zone 3A. In terms of natural channel conditions, this zone is on the downstream face near the waterway end of the embankment. It is unlikely that the influence of trees and brush between this point and zone 2 would ordinarily be significant.

Mr. Lin's remarks concerning contradictory definitions of C appear to be based on a misunderstanding of the purpose of the dimensional analysis leading to Eq. 6*b*. In the section entitled "Simple Constrictions in Rectangular Open Channels: Selection of the Independent Variables" the resistance characteristics of the upstream channel are recognized, but they are omitted from the dimensional analysis in anticipation of their subsequent treatment in the discharge equation. Thus, the second dimensionless group in Eq. 6*b* is described as a ratio that is "proportional to a coefficient of discharge." It is, in fact, simply an indication of the existence of a general parameter having the form of a Euler number. It was the purpose of the section following Eq. 6 to formulate the coefficient of discharge expressed by Eqs. 10 and 12. A brief justification of the writers' method of accounting for h_f and $\alpha_1 V^2{}_1/2g$ is contained in the paragraph following Eq. 9.

Mr. Lin describes some of the complications that result from the application of the one-dimensional energy and momentum equations to a section in which the pressure is nonhydrostatically distributed. It is usually possible to avoid these complications by applying the equations to zones of parallel flow; that is, to sections in which the normal acceleration—and, therefore, the normal piezometric-head gradient—is zero. This was accomplished in the writers' study by defining section 2 quite arbitrarily as the vena contracta. Actually, of course, this qualification is largely academic. Regardless of where section 2 is located, both α_2 and β_2 ordinarily will be functions of the independent variables upon which the experimentally determined values of C depend. It follows that the influence of a nonhydrostatic pressure distribution, in addition to the influence of a nonuniform velocity distribution in the constricted section, is contained in the coefficient of discharge defined by Eq. 13.

The writers are grateful for Mr. Blasidell's recognition of the large amount of work and the many tests required to establish empirical solutions for complex fluid phenomena. Problems involving open channels, particularly natural channels, typically depend on many independent variables. It is seldom possible to consider all the variables in a laboratory investigation. The decision to limit the experiments to values of m greater than 0.20 was one of the consequences of the necessity to restrict the scope of the study to the conditions most frequently encountered in practice. Nevertheless, it should be recognized that the tests were conducted at a relatively small scale and therefore that values of m less than 0.20 would result in very small changes in water-surface level and quantities such as Δh would be subject to large experimental errors. The writers agree with Mr. Blaisdell that, from the viewpoint of the researcher, so-called "practical" limits are often frustrating.

Much of Mr. Izzard's discussion is concerned with the determination of the backwater due to an open-channel constriction. This is one of the objectives of the comprehensive investigation in which the writers are engaged. An analysis and much additional experimental data pertaining specifically to that

problem are therefore contained in a subsequent paper.[13]

Mr. Izzard is concerned about the fact that, in all the writers' tests, the bottom of the experimental flume was horizontal. The significance of this limitation is that a small part of the change in depth in a level flume is due to the acceleration which results from the unbalanced boundary resistance force. For the conditions of this investigation the influence of bottom slope is believed to be insignificant. Mr. Izzard also proposes a simplification in the method of computing h_f. In his words, "All the energy losses attributable to the geometry of the constriction would therefore be included in the discharge coefficient." Actually, he would be adding one variable, the conveyance of the constricted waterway, to those already involved in the evaluation of the discharge coefficient. That this variable has been effectively removed in the writers' solution is demonstrated by the lack of correlation between C and n throughout the full range of tests reported. In connection with both of Mr. Izzard's criticisms, it should be observed that the contracted-opening method of determining the discharge in natural channels is not feasible when the boundary resistance loss, h_f, is a large part of Δh.

Mr. Posey deplores the omission of certain natural channel variables, including constriction roughness as well as borrow pit, drainage ditch, and right-of-way characteristics. In a subsequent report[32] many variables were considered in addition to those involved in the present study. These variables, all concerned with natural bridge waterways, resulted from sloping embankments, a variety of abutment forms, piles and piers, and partial bridge submergence. Thus, of the "almost unlimited" number of variables acknowledged in the "Introduction," a large number have been considered. It is possible that experience in the application of the proposed computation procedure will indicate the necessity for experiments involving additional variables. In the meantime, growing evidence of the successful application of the writers' data to the computation of flood flows in natural waterways is encouraging.

Discussion of
"THE PRESENT STATUS OF RESEARCH ON SEDIMENT TRANSPORT"

by Ning Chien
(Proc. Paper 565)

HSIN-KUAN LIU.[1]—To make a brief summary of what has been studied and found in the field of sediment transport is not an easy task, because numerous papers dealing with the mechanics of this subject have been published. It is even more difficult to bring conclusions from various studies because of the widely spread opinions among them. The author certainly has done a timely summary which helps the advancement of research in the field of sediment transport.

In discussing the initiation of ripples, the author introduced the Writer's result as shown in Fig. 2 Curve II (Fig. 7 in this discussion). The result is based upon the assumption that ripples are due to instability of the interface (or boundary) between the flow and the movable bed. The author argued that since the sediment rate is not large enough to create any difference in density between the clear water and the sediment-laden water, ripples cannot be associated with the instability at the interface of two flows of different densities. The writer suggests the following as a possible explanation of the mechanics of ripple formation.

Due to difference in velocity between the flow and the movable bed, there exists an interface having a high velocity gradient. Furthermore, such an interface is very unstable because of this high-velocity gradient. Since the bed is movable, the bed configuration is readily affected by temporal variations of flow pattern, if the local flow velocity is large enough to transport the bed material. The next question is what causes this temporal variation of flow pattern which results in the instability of the interface. This question can be answered, at least in part, by the analysis which Rouse used in explaining the instability of flow at a hypothetical surface of discontinuity.(94) At any instant the streamline at the interface may deviate from a straight line due to a disturbance which likely exists in the flow. The effect of the disturbance upon the flow pattern is dependent upon the instability of the interface (or boundary). According to the flow net, the local velocity will decrease as the neighboring streamlines diverge, and increase as they converge. If the local velocity of the flow is large enough to transport the sediment, the periodic change in velocity along the interface will result in a change of bed configuration. Exner (22) has established a differential equation for sediment transport in two-dimensional flow. That is Eq. 40 indicates that the variation

$$\frac{\partial \eta}{\partial t} + K\frac{\partial V}{\partial x} = 0 \qquad (40)$$

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where, η is the bed elevation,
t is time,
K is a factor relating sediment discharge to flow velocity,
v is the flow velocity near the bed and is a function of distance along the flow direction,
x is the distance in the downstream direction.

of bed elevation is due to velocity variation with respect to distance in the flow direction.

An index indicating the instability of the interface can be expressed by a dimensionless parameter, $\frac{DV_*}{\nu}$, which is a special form of Reynolds number. On the one hand the greater the shear velocity, which means a higher velocity gradient at the interface, the more pronounced will be the instability of the interface; and the larger the sediment size, the more pronounced will be the tendency to produce eddies. On the other hand, the higher the viscosity, the more the internal shear will tend to oppose further differences in velocity.

The capability of the flow to transport the sediment and change the bed configuration is dependent upon the scouring force of the flow and the resistance force of the sediment. The scouring force can be expressed as (24)

$$F_1 = C_1 \frac{1}{4} \pi d^2 \left(\frac{1}{2} \rho_f V_*^2\right) \quad (41)$$

the resistance force of the sediment to the flow is (24)

$$F_2 = C_2 \frac{1}{4} \pi d^2 \frac{1}{2} \left(\rho_f V_s^2\right) \quad (42)$$

where F_1 is the erosive force of the flow,
F_2 is the resistance force of the sediment,
D is the sediment size,
ρ_f is the fluid density,
V_* is the shear velocity of the flow,
V_s is the fall velocity of the sediment,
ν is the dynamic viscosity of the fluid,
C_1, C_2 are functions of $\frac{DV_*}{\nu}$ and shape factor of the sediment.

Laboratory experiments indicate that ripples occur shortly after the beginning of bed-load movement. Therefore, it can be assumed without serious error that, at the moment ripples occur, the scouring force is equal to the critical scouring force.

$$F_1 = F_2 \quad (43)$$

so that

$$\frac{V_*}{V_s} = \text{function}\left(\frac{DV_*}{\nu}, \text{shape factor}\right) \quad (44)$$

If shape factor is considered to be of secondary importance, then Eq. 44 can be written as

$$\frac{V_*}{V_s} = \text{function}\left(\frac{DV_*}{\nu}\right) \qquad (45)$$

At the moment ripples occur, two conditions must be satisfied: (a) the flow must be strong enough to transport the sediment; (b) the interface between the flow and the movable bed must become unstable. It should be noted that $\frac{DV_*}{\nu}$ as mentioned above is a parameter which serves two functions: (a) it indicates the instability of the interface; (b) it correlates the flow velocity at the grain level to the shear velocity in accordance with Karman-Prandtl theory, thereby defining the scouring force. By plotting data which are the average between smooth bed and first appearance of ripples in terms of $\frac{V_*}{V_s}$ versus $\frac{DV_*}{\nu}$, Fig. 7 was obtained. Conditions (a) and (b) are both satisfied when data pertaining to the flow and the sediment fall above the curve. The scatter of the data is due to such causes as error in measurements, difficulties of defining the first appearance of ripples, shape factor, and effect of mixture.

The author pointed out that there are many bed-load formulas today. However, their application to engineering problem is limited because of various reasons. The ones such as Einstein's, Kalinske', and Meyer-Peter's are sound in theory, yet too complicated for many hydraulic engineers to understand. Furthermore, since there is no way to measure the actual bed-load in a large river, their applicability to a large river is uncertain. Many investigators are still searching for a better bed-load formula and also method of measuring bed-load.

The writer's experiment on sand wave movement, as will be explained below, seems to indicate the possibility of obtaining a type of bed-load formula in terms of sand wave motion, and of measuring the amount of bed-load transported by investigating the sand wave motion.

In performing the experiments the writer used a recirculating, open-channel, laboratory flume 40 ft long by 1 ft wide by 2 ft deep. There were two 5-ft glass sections on one side and one 5-ft section on the other to facilitate observation. Water was supplied from a floor-level sump of 8 ft by 2.5 ft by 2.5 ft through a centrifugal pump to a 4-in. feed pipe. Flow was measured through an orifice meter and returned to the supply sump over an adjustable tailgate at the downstream end of the flume. Sand was supplied at the upstream end of the flume by means of an elevator. The elevator was operated by a variable speed motor, chain, and sprocket-wheel connected to a screw-lift mechanism. Ottawa 20-30 sand was used as sediment. Sand transported to the downstream end of the flume was trapped in a bucket and weighed to verify the rate of sediment feed. The experiments were done in the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota. The whole work was part of the writer's doctoral dissertation under the guidance of Professor Lorenz G. Straub.

The purpose of the experiments was to determine the quantitative relationship between sand wave motion and bed-load transport. It was found that approximately 100 per cent of the bed-load is transported in the form of sand waves, except when there is an appreciable amount of saltation load in the stream. In these experiments, the discharge was kept constant at 0.4 cfs. The rate of sediment feed ranged from 600, 300, 150, 75, to 38.3 lbs per hour. The bed slope was varied from 0.0115 to 0.002. Mean velocity of the flow varied

from 2.3 to 1.5 fps. Depth of flow varied from 0.174 ft to 0.267 ft. Froude number ranged from 0.97 to 0.512.

The velocity of the sand waves was measured by two methods; (a) counting the number of waves passing a certain station per unit time and multiplying this by the wave length; (b) measuring the time required for a certain wave to travel a certain distance. The wave length was obtained by averaging the total length of several waves. It was found, as shown in Fig. 8, that the velocity of the sand waves for the case where $q = 0.40$ cfs with Ottawa 20-30 sand ($d = 0.690$ mm.)

$$c = 0.0008\,V^{4.7} \tag{46}$$

where c is the sand wave velocity and V is the mean velocity of flow. Inasmuch as the wave velocity varies with a large exponential power of the mean velocity of the flow, the wave velocity is highly sensitive to change of the flow velocity.

In studying the relationship between sand wave motion and rate of bed-load transport, the bed profile was measured by a point gage with an electric device attached. As the point touched the wetted bed surface, a red light appeared on the gage. The depth of the flow was the vertical distance between water surface and the mean bed elevation. In order to compute the bulk volume of the sand wave during each run, five bed profiles were taken along the flume. The lower boundary of the sand waves was considered to be the line joining the lowest points of the troughs. The average volume per sand wave was that of the total volume of sand waves above the lower boundary divided by the number of waves. The volume of sand waves per unit width was measured from the bed profiles by a planimeter. Profiles of Run 2-2 are shown in Fig. 9. Results of the experiments are listed in Table I. The last two lines in Table I show both the actual and the computed amount of sediment transported. It indicates that bed-load is entirely in wave forms. Translation of sediment along the bed is negligible. The computed amount of sediment in Run 2-1 is considerably below the actual amount of sediment introduced. This can be explained by the fact that the sediment feed was heavy and a considerable amount of it was moving as saltation load.

Results previously shown suggest that a formula for bed-load transport in terms of sand wave motion, sediment properties, and flow properties might possibly be obtained by extensive experiments. The formula, if obtained in the future, could be used to verify existing bed-load formulas. Field measurements of rate of bed-load transport could be made by observing the sand wave velocity during flood stage, and the bed profile could be taken by echo sounding devices.

REFERENCE

94. "Elementary Mechanics of Fluids" by Hunter Rouse, John Wiley & Sons, Inc., New York, 1947.

TABLE I

Summary of Experimental Results of Properties of Sand Waves

Run No.		2-1	2-2	2-3	2-4	2-5
discharge	cfs	0.4	0.4	0.4	0.4	0.4
depth	ft	0.174	0.190	0.215	0.236	0.267
velocity	fps	2.3	2.1	1.86	1.70	1.50
slope		0.0115	0.0086	0.0054	0.0032	0.002
R_w	ft	0.035	0.051	0.06	0.077	0.092
R_b	ft	0.162	0.171	0.189	0.189	0.218
F_r		0.972	0.848	0.707	0.615	0.512
amplitude	ft	0.089	0.078	0.059	0.037	0.053
wave length	ft	0.92	0.835	0.834	0.715	1.58
wave vel.	fps	0.0455	0.0236	0.014	0.0093	0.0051
t^o F		72	77	73	76	75
T_b	#/ft	0.116	0.092	0.064	0.038	0.027
V_*	fps	0.245	0.22	0.18	0.14	0.12
Sedi. supply	#/hr	600	300	150	75	38.3
Sedi. Comput.	#/hr	440	297	157	72.5	29.6

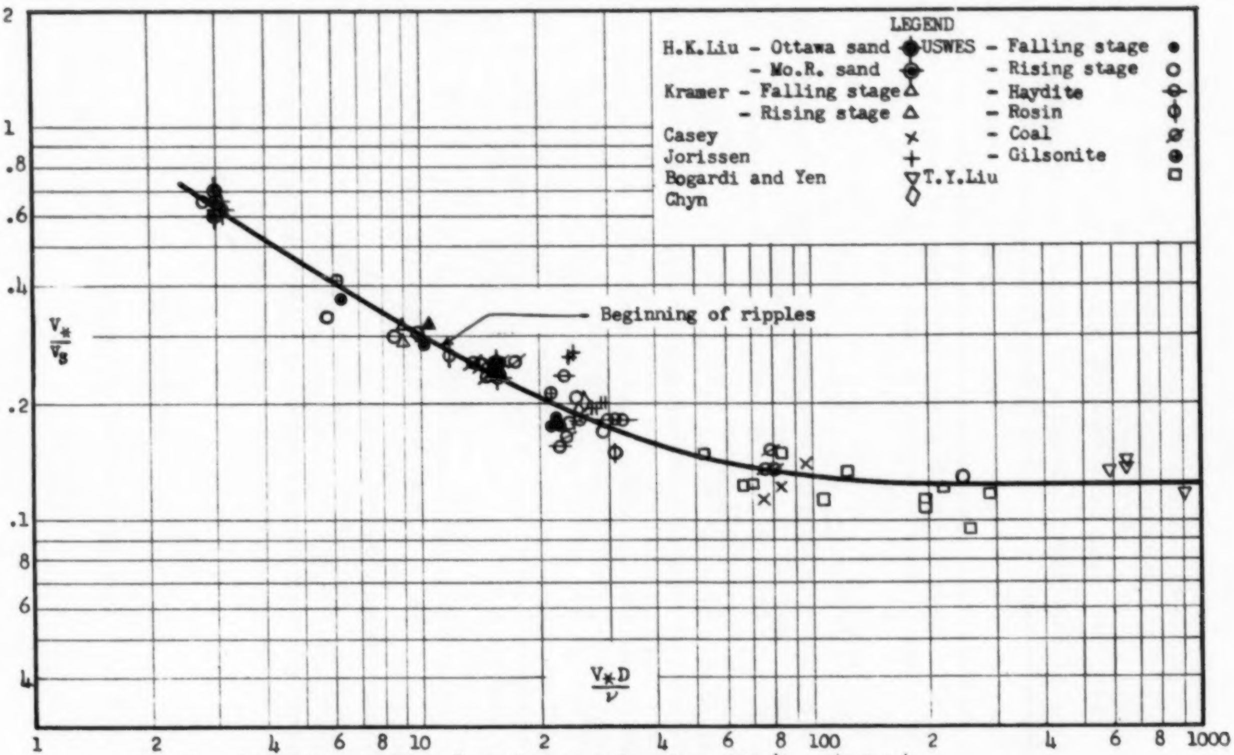


Fig. 7 - Criterion of Ripple Formation as a Function of V_*/V_s and V_*D/ν

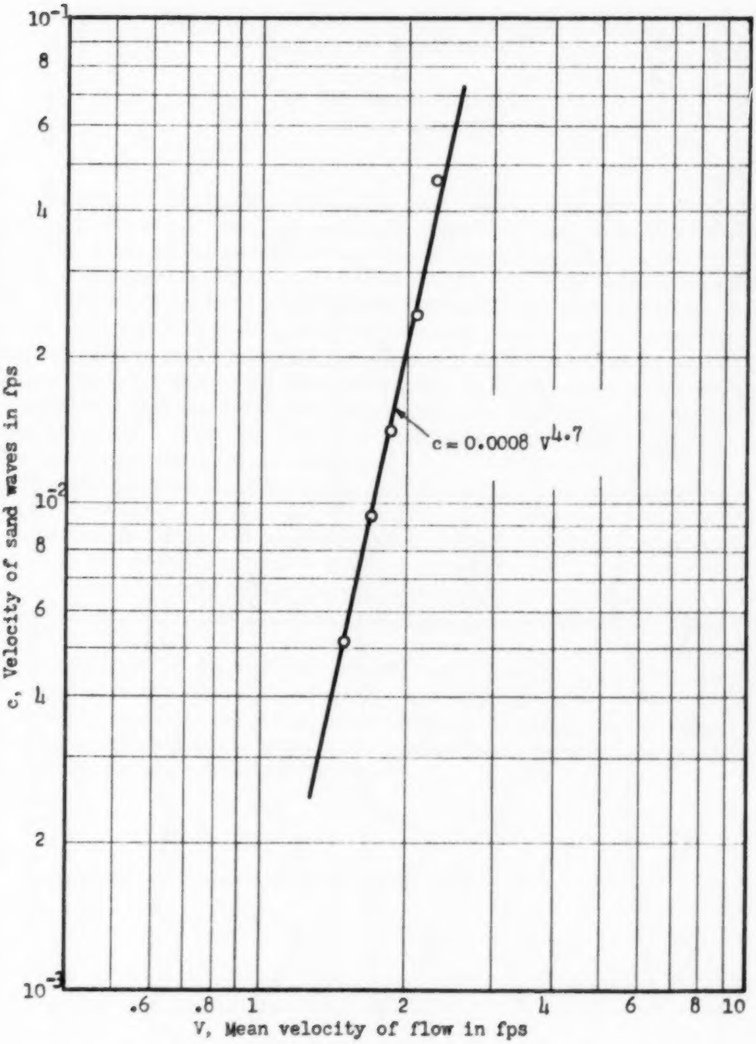
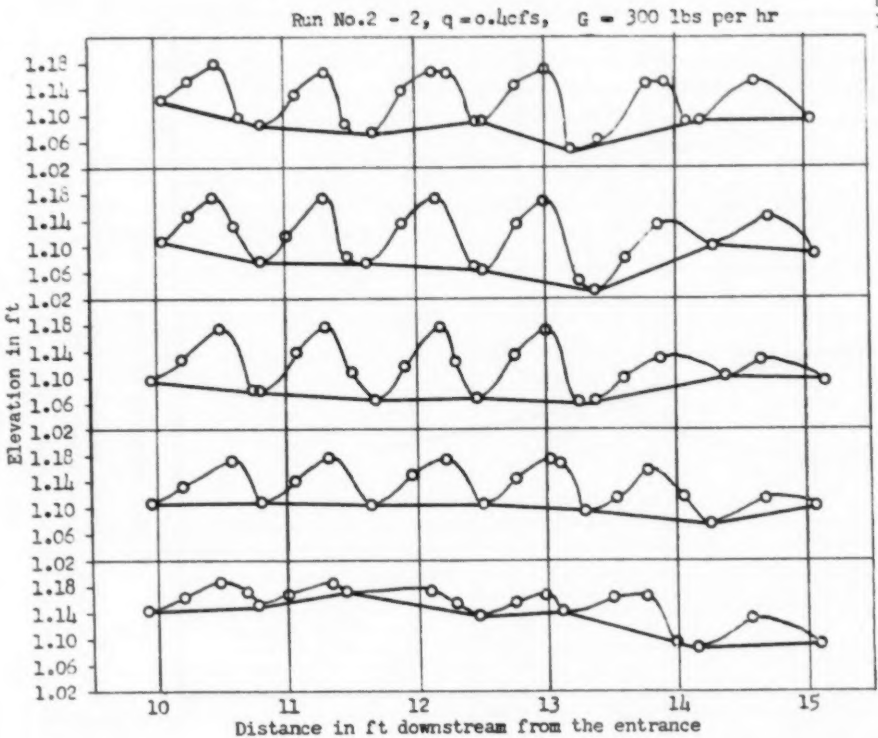


Fig. 8 - Relationship between Velocity of Sand Wave and Velocity of Flow



Distance from left side wall in inches	Average area per wave in sq ft
2	0.032
4	0.036
6	0.035
8	0.030
10	0.017

weighted area = 0.029 ft^2

Fig. 9 - Example of Plot of Sand Wave Profiles Taken between St.10 and St.15

Discussion of
"MECHANICS OF STREAMS WITH MOVABLE
BEDS OF FINE SAND"

by Norman H. Brooks
(Proc. Paper 668)

T. BLENCH,[1] M. ASCE.—From the description of experiments the writer believes that a steady state of sediment movement had not been reached and that, even if it had, no quantitatively useful results could have been obtained with the suspended and bed loads measured as one. The test for equilibrium ("The two sets of profiles were practically identical, indicating that the equilibrium achieved was stable, well defined, and reproducible.") seems to be no more than a test of the fact that the same conditions applied for the same time produce the same degree of disequilibrium. It is not surprising therefore that "the data obtained are not sufficiently extensive to permit establishing quantitative formulas." Why they should be considered sufficient to destroy two well-established theories, of which regime theory is one, is not clear.

The writer draws attention to the following misunderstandings of regime theory. First that it is his own, whereas he has always been at pains to attribute it to Gerald Lacey. Second that it avoids consideration of shear, whereas, in the way he presents it, gdS is brought explicitly into the regime slope formula. Third that basic data were not made available, whereas the quoted reference (15) gives its own references 2, 3, 6, 7, 9, 10 abounding with data. Fourth that it is based on an erroneous assumption concerning suspended load (vide SUMMARY OF CONCLUSIONS) whereas it makes no assumption about suspended load and offers no means for dealing with it; it is essentially a bed-load theory. Actually the theory consists of explaining observed facts of canal self-adjustment that were found to be represented by certain formulas; the formulas do not depend on the theory.

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PROCEEDINGS PAPERS

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APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)[c], 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)[c], 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)[c], 700(PO), 701(ST)[c].

JUNE: 702(HW), 703(HW), 704(HW)[c], 705(IR), 706(IR), 707(IR), 708(IR), 709(HY)[c], 710(CP), 711(CP), 712(CP), 713(CP)[c], 714(HY), 715(HY), 716(HY), 717(HY), 718(SM)[c], 719(HY)[c], 720(AT), 721(AT), 722(SU), 723(WW), 724(WW), 725(WW), 726(WW)[c], 727(WW), 728(IR), 729(IR), 730(SU)[c], 731(SU).

JULY: 732(ST), 733(ST), 734(ST), 735(ST), 736(ST), 737(PO), 738(PO), 739(PO), 740(PO), 741(PO), 742(PO), 743(HY), 744(HY), 745(HY), 746(HY), 747(HY), 748(HY)[c], 749(SA), 750(SA), 751(SA), 752(SA)[c], 753(SM), 754(SM), 755(SM), 756(SM), 757(SM), 758(CO)[c], 759(SM)[c], 760(WW)[c].

c. Discussion of several papers, grouped by Divisions.

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